

ANALYTICAL MODELING OF HYSTERETIC SDOF SYSTEMS CONSIDERING POST-PEAK BEHAVIOR AND ENGINEERING IMPLICATIONS

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Abstract

A theory-based analytical hysteretic model is presented in the study to include post-peak behavior into nonlinear dynamic analysis. Unlike other rule-based models, the proposed mathematical form can be readily embedded into existing in-house and/or commercial software with no excessive coding efforts. On the other hand, the introduction of performance-based earthquake engineering into the seismic design documents has indicated the necessity of considering post-peak behavior of structural systems into nonlinear dynamic analysis especially at the hazard level of very rare events such as 2% exceedance probability in 50 years.

INTRODUCTION

Performance-based seismic design philosophy has been recently incorporated into new generation code documents. An appropriate performance measure within the framework of performance-based seismic engineering should be capable of representing both new-designed and existing structures. As such, a conceptual Performance Matrix has been recently proposed in Pampanin et al. (2003), using both maximum and residual deformations as performance evaluation indices. The reasons to evolve a currently popular and relatively simple evaluation method into a more knowledge-demanding performance evaluation method is based on an understanding of the following aspects:

1. The consideration of near-fault motions with severe directivity velocity pulses and/or static displacement fling could lead to permanent deformation of the structural system when highly nonlinear structural behavior occurs. Moreover, a 2% in 50 years hazard level usually make an ordinary structure almost reach its collapse state.
2. Buildings designed to older code documents are susceptible to severe damage or may even collapse during a severe seismic event. This is especially true for the observed low-cycle collapse of reinforced concrete frame buildings with light transverse column reinforcement during the 1999

Chi-Chi earthquake.

3. Modern advanced structural systems, in contrast to traditional systems, may be capable of re-centering itself back to the original position after earthquakes.

The former two aspects are the main concerns of this study, while one may refer to Pampanin et al. (2003) regarding the 3rd aspect on modern advanced structural systems. Modern advanced structural systems with re-centering device have inspired the use of residual deformation as an additional performance evaluation index in the near future. As we know, the current performance evaluation method is based on one or multiple structural response indices including maximum drift ratio (or, ductility) and cumulative inelastic energy dissipation, which are known to be able to fully characterize performance levels for systems where the main concern is to avoid collapse, but are unable to characterize the performance level of some modern structural systems (e.g., self centering frame buildings) where the structural integrity is not at risk during seismic attack. Besides, in the three aforementioned cases maximum response itself may not be able to fully represent the final status of the structure. As such, an independent scale of residual-deformation based performance measure can be effectively combined with the existing performance measure based on maximum response (or, cumulative damage) to form a more general performance domain. For different seismic intensity levels it would result in a full 3-dimensional performance domain, which is schematically described in Fig. 1 (Pampanin et al., 2003).

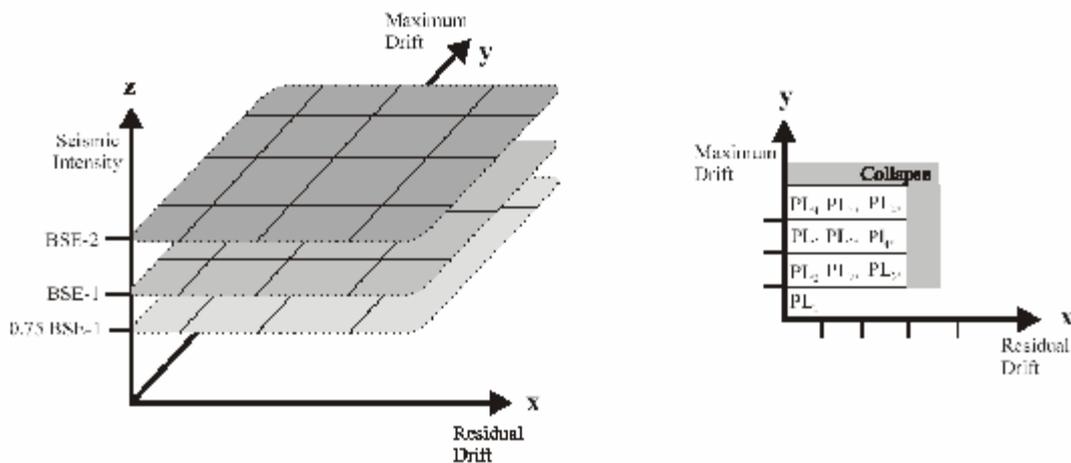


Fig. 1: Framework for Residual-Maximum Performance Based Approach: Performance Matrix (source: Pampanin et al. 2003).

MATHEMATICAL FORMULATION OF HYSTERETIC MODEL WITH CONSIDERATION OF POST-PEAK BEHAVIOR

The governing equation of motion for an SDOF oscillator demonstrating hysteretic behavior can be expressed as:

$$m\ddot{x} + c\dot{x} + a_n kx + k \sum_{i=1}^{n-1} a_i u_i = -m\ddot{x}_g \quad (1)$$

where m is mass, c is viscous damping coefficient, k is initial stiffness of the system, a_i is

post-to-preyield stiffness ratio (or, strain hardening ratio), u_i is auxiliary state variables, x_g is ground displacement, and x is relative displacement of the SDOF oscillator with respect to the ground. Dots indicate time derivative. The hysteretic models found in the literature to describe nonlinear behavior of deteriorating structural systems under cyclic loading can be generally categorized into three groups: (1) smooth hysteretic model, e.g., Wen, 1976, Baber and Noori, 1985, Wang and Wen, 1998, Sivaselvan and Reinhorn, 2000, etc.; (2) rule-based polygonal hysteretic model, e.g., Park et al. 1987, Shi, 1997, Elwood, 2002, Ibarra et al., 2002, etc.; (3) theory-based piecewise linear hysteretic model, e.g., Mostaghel, 1999, etc. In this study, a new theory-based piecewise linear model is proposed, and it can be used to define constitutive relationship between stress and strain, force and displacement, moment and curvature, or moment and rotation, depending on the applications as long as the quantity and quality of experimental results are sufficient for the determination of the values of key model parameters. The SDOF hysteretic system in Fig. 2 contains one linear spring, the deformation of which is represented by x , and a slider-spring element with a frictional surface of variable Coulomb damping coefficient such that the slider-spring will start to slip at a certain force level (e.g., $k \cdot u_y$ at first yield). The slip may accelerate when the friction coefficient reduces to a lower level because of a decrease in the interlocking force between internal particles of the system, which, in the case of RC columns, physically means major shear cracks have fully developed. Such post-peak behavior is commonly observed in low-confinement RC columns, pre-Northridge steel connections, and wood shear wall system. To describe such a phenomenon, unknown u representing the deformation of the spring connected to the slider can be expressed as the following mathematical form:

$$\dot{u} = \dot{x} \cdot \left\{ \begin{array}{l} \left[\begin{array}{l} \mathbf{f}_k \cdot \bar{M}(u - l_p \cdot \mathbf{f}_l \cdot \mathbf{d}_y^+) \cdot \bar{M}(x) + \mathbf{f}_k \cdot \bar{M}(u - \mathbf{f}_l \cdot \mathbf{d}_y^+) \cdot \bar{N}(x) \\ + (-\mathbf{f}_c^+ - \mathbf{f}_k) \cdot \bar{N}(x - \mathbf{d}_u^+) \cdot \bar{M}(x - \mathbf{d}_r^+) \cdot N(u) \cdot N(x - \mathbf{d}_{FS}^+) \\ + (-\mathbf{f}_r^+ - \mathbf{f}_k) \cdot \bar{N}(x - \mathbf{d}_r^+) \cdot N(u) \cdot N(x - \mathbf{d}_{FS}^+) \end{array} \right] \\ + M(\dot{x}) \cdot \left[\begin{array}{l} \mathbf{f}_k \cdot N(u + l_p \cdot \mathbf{f}_l \cdot \mathbf{d}_y^-) \cdot N(x) + \mathbf{f}_k \cdot N(u + \mathbf{f}_l \cdot \mathbf{d}_y^-) \cdot M(x) \\ + (-\mathbf{f}_c^- - \mathbf{f}_k) \cdot M(x + \mathbf{d}_u^-) \cdot N(x + \mathbf{d}_r^-) \cdot \bar{M}(u) \cdot N(-x - \mathbf{d}_{FS}^-) \\ + (-\mathbf{f}_r^- - \mathbf{f}_k) \cdot M(x + \mathbf{d}_r^-) \cdot \bar{M}(u) \cdot N(-x - \mathbf{d}_{FS}^-) \end{array} \right] \end{array} \right\} \quad (2)$$

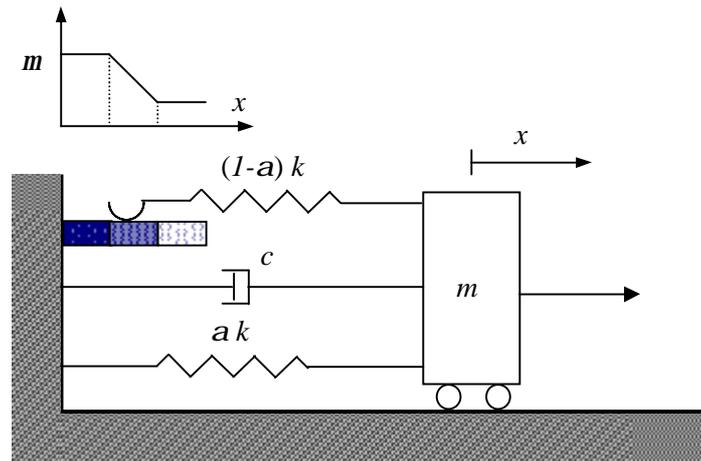


Fig. 2: Schematic representation of SDOF hysteretic system with consideration of post-peak behavior

in which, $N(x) = H(x)$; $M(x) = 1 - H(x)$; $\bar{N}(x) = 1 - H(-x)$; $\bar{M}(x) = H(-x)$, and $H(x)$ is the Heaviside's unit step function. The constant I_p , which takes a value between 0 and 1, denotes the resistance ratio and represents pinching of a hysteretic loop due to unequal strengths. There is no pinching in a structural component if I_p is assumed to be 1. d_y is the yield displacement of the system; d_u is the displacement at which the system reaches its ultimate strength and the friction coefficient of the slider will start to decrease; d_r is the displacement at which the system reaches its residual strength; d_{FS} defines the failure surface in Fig. 3 and represents the corresponding deformation of the linear spring when the slider-spring reaches a certain deformation at a time step. The positive and negative signs in the superscript indicate asymmetric material property may exist under compression and tension. The failure surface may migrate to a lower level of material strength according to a prescribed flow rule when dissipated hysteretic energy accumulates with time. In addition,

$$\text{Stiffness-degradation function} \quad f_k = \frac{1}{1 + I_k h(t)} \quad (3)$$

$$\text{Load-deterioration function} \quad f_l = \frac{1}{1 + I_l h(t)} \quad (4)$$

$$\text{Post-peak stiffness function} \quad f_c = \frac{(1-r)d_y}{d_r - d_u} \quad (5)$$

And, $h(t)$ denotes the hysteretic energy absorbed by the system. A multilinear system with more than one slider-spring element may also be used to obtain a smoother hysteretic loop. For brevity, one slider-spring model is used in the study since such an approximation is commonly used in practice, is usually accurate enough in most cases, and can also lessen computational efforts. Unlike Bouc-Wen hysteretic model, the physical meanings of the proposed model parameters are self-evident, so the parameter values can be determined with no need to go through complicated nonlinear regression procedures.

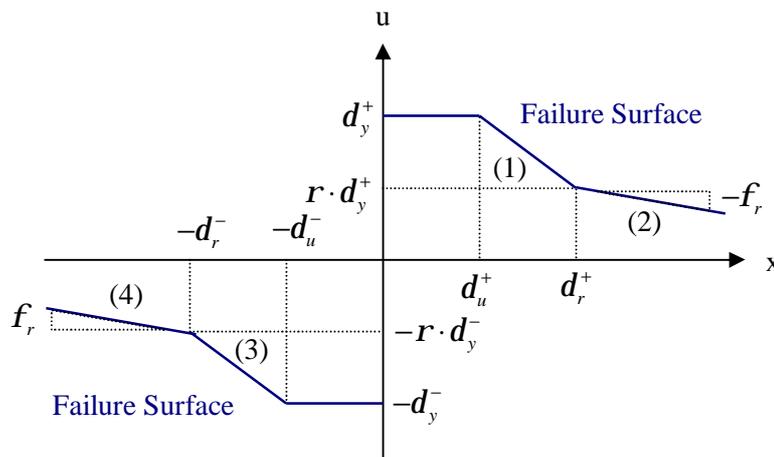


Fig. 3: schematic representation of failure surface

Fig. 4 demonstrates several numerical hysteretic models having negative post-peak stiffness in the left column and relevant experimental results showing the same characteristics are given in the right column: (1) Bilinear hysteresis with capping (top left), and a welded haunch steel moment connection taken from Uang et al. (2000) (top right), (2) Pinching with collapse (middle left), and a concrete shear wall taken from Oh et al. (2002) (middle right), (3) Pinching with collapse (bottom left), and a low-ductility RC column taken from Elwood (2002) (bottom right).

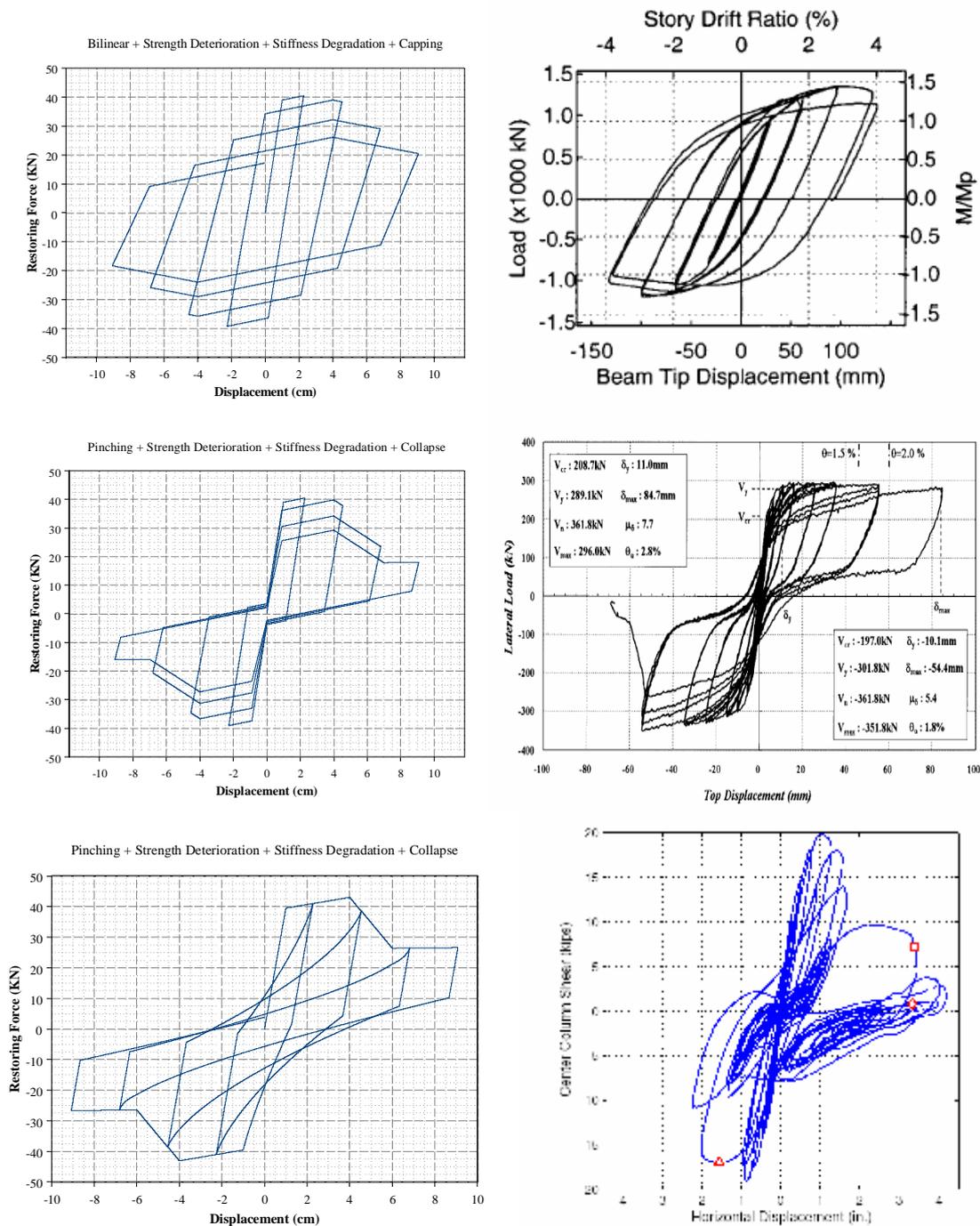


Fig. 4: Numerical hysteretic models having post-peak negative stiffness (left column) and experimental results showing similar characteristics (right column).

PERFORMANCE EVALUATION METHOD CONSIDERING PEAK AND RESIDUAL DEFORMATIONS

A preliminary study has been conducted by the authors, and numerical results show that collapse consideration is essential in order to implement the newly proposed Performance Matrix in the future design codes. Experience shows that peak and residual deformations can be predicted with accuracy

only if post-peak negative stiffness and collapse are taken into account in structural response analysis. Collapse analysis on a 12-story RC building is discussed in the following.

Description of 12-story RC Model Building

A 12-story RC building with a natural period of 1.61 sec was designed in Liao and Wang (1999). The building has a 25cm yield displacement according to static pushover analysis and a 10% pre-to-postyield stiffness ratio. 5% viscous damping of critical is assumed. To perform collapse analysis on the building, an equivalent SDOF system using base shear formulation suggested by Collins et al. (1996) is used. Reliable estimate of global (roof) drift ratio of the building is expected using the suggested procedure. For non-deteriorating bilinear system, $f_k = 1$, $f_l = 1$, $I_k = 0$, $I_l = 0$ and $I_p = 1$. $I_p = 0.3$ is assumed for pinching system; $I_k = 1$ and $I_l = 0.1$ for deteriorating system. $r = 0.25$ is assumed for residual strength. When collapse is a concern, $d_u = 4d_y$, $d_r = 6d_y$, and $f_r = 0.1$ is assumed for the system.

Suites of Uniform Hazard Ground Motions

Design spectra corresponding to 2 hazard levels of Design and Maximum Considered Earthquakes from the Taiwanese seismic design code are constructed in Fig. 5 to represent the base shear coefficients of the Xinyi district of Taipei basin. According to probabilistic seismic hazard analysis results (Chien, 2003), an intermediate hazard level of 5% exceedance probability in 50 years is added to our response analyses. To get 10 uniform hazard earthquake motions at each hazard level, TSMIP array data in Taipei basin since 1994 are selected to match the design spectra in a wide range of periods. Important characteristics of these selected motions are summarized in Tables 1 through 3. The scaling factors are preferentially no larger than 12.61, which imply that no small magnitude earthquakes are taken to represent much higher hazard levels of large magnitude earthquakes. Due to the limitation of data obtained from the field, it is observed that most of the selected motions are from the 1999 Chi-Chi earthquake and the March 31, 2003 Hualien earthquake. Median, 16- and 84-percentile spectra of the 10 selected motions are plotted against the target design spectra for comparison (Fig. 5).

Table 1: Important characteristics of 10 earthquake motions in the 10% in 50 years hazard level

Date (GMT)	M_L	Focal Depth (km)	Station ID	Duration (sec)	PGA (g)	Scaling Factor
1995/6/25	6.5	39.9	TAP008	92	0.050	8.47
1999/9/20	7.3	8.0	TAP	60	0.042	3.28
1999/9/20	7.3	8.0	TAP005	134	0.083	1.74
1999/9/20	7.3	8.0	TAP007	134	0.071	2.42
1999/9/20	7.3	8.0	TAP083	120	0.035	5.23
1999/9/20	7.3	8.0	TAP051	90	0.071	3.25
2000/9/10	6.2	17.7	TAP050	70	0.027	8.28
2002/3/31	6.8	13.8	TAP094	95	0.045	4.20
2002/3/31	6.8	13.8	TAP044	75	0.062	2.50
2002/3/31	6.8	13.8	TAP015	90	0.130	1.50

Table 2: Important characteristics of 10 earthquake motions in the 5% in 50 years hazard level

Date (GMT)	M _L	Focal Depth (km)	Station ID	Duration (sec)	PGA (g)	Scaling Factor
1995/6/25	6.5	39.9	TAP008	92	0.050	9.55
1999/9/20	7.3	8.0	TAP	60	0.042	3.70
1999/9/20	7.3	8.0	TAP083	120	0.035	5.90
1999/9/20	7.3	8.0	TAP047	87	0.079	4.24
2002/3/31	6.8	13.8	TAP075	76	0.056	6.67
2002/3/31	6.8	13.8	TAP044	75	0.040	4.10
2002/3/31	6.8	13.8	TAP044	75	0.062	2.82
2002/3/31	6.8	13.8	TAP040	79	0.052	4.25
2002/3/31	6.8	13.8	TAP110	87	0.065	2.90
2002/3/31	6.8	13.8	TAP015	90	0.130	1.70

Table 3: Important characteristics of 10 earthquake motions in the 2% in 50 years hazard level

Date (GMT)	M _L	Focal Depth (km)	Station ID	Duration (sec)	PGA (g)	Scaling Factor
1995/6/25	6.5	39.9	TAP008	92	0.050	12.61
1999/9/20	7.3	8.0	TAP100	144	0.065	4.32
1999/9/20	7.3	8.0	TAP083	120	0.035	7.79
1999/9/20	7.3	8.0	TAP051	90	0.071	4.84
1999/9/10	6.2	17.7	TAP050	70	0.027	12.33
2002/3/31	6.8	13.8	TAP094	95	0.045	6.25
2002/3/31	6.8	13.8	TAP044	75	0.040	5.42
2002/3/31	6.8	13.8	TAP044	75	0.062	3.73
2002/3/31	6.8	13.8	TAP041	84	0.076	4.73
2002/3/31	6.8	13.8	TAP015	90	0.130	2.24

Nonlinear Time History Analysis and Engineering Implications

Nonlinear time history analysis is performed on the equivalent SDOF system to yield estimates of maximum and residual roof drift ratios of the 12-story RC building using the 5th order Cash-Karp Runge-Kutta method to implement adaptive time stepping algorithm. Numerical results are given in Fig. 6 in the format of Performance Matrix. Although not significant, the bilinear system generally has a lower level of maximum drift ratios, but its residual drift ratio is slightly higher. This is because in this study an identical level of strength deterioration and stiffness degradation is assumed for bilinear and pinching systems. Comparisons are also made in system responses with and without consideration of collapse mechanism. It is observed that response estimates at the 10% in 50 years level coincide for both cases since collapse, mostly, does not occur. However, as long as 2% in 50 years hazard level is of concern, the response estimate will strongly depend on whether collapse is taken into consideration. This observation surely has very important implications in seismic design, and performance evaluation of structural systems under seismic excitation. If the use of Performance Matrix is a necessity in the framework for performance-based earthquake engineering, then the consideration of collapse in

dynamic analysis will help map a structure's performance into its corresponding Performance Level cells, as shown in Fig. 1, with confidence. In passing, it is noted that the traditional analysis approach usually provides only information on whether the building collapses according to engineering judgment and experience, but a confident estimate of maximum and residual drift is not possible.

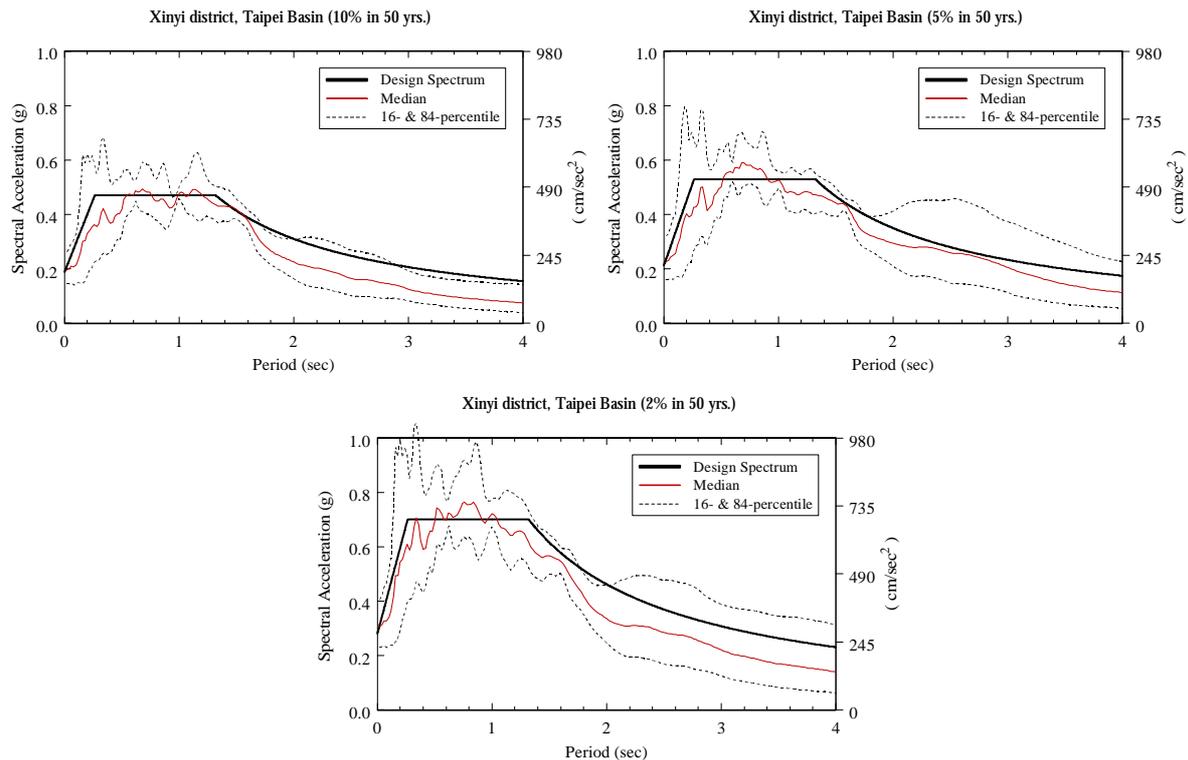


Fig. 5: Design spectra compared with median, 16- and 84-percentile spectra of 10 selected ground motions at 10%, 5% and 2% in 50 years hazard levels.

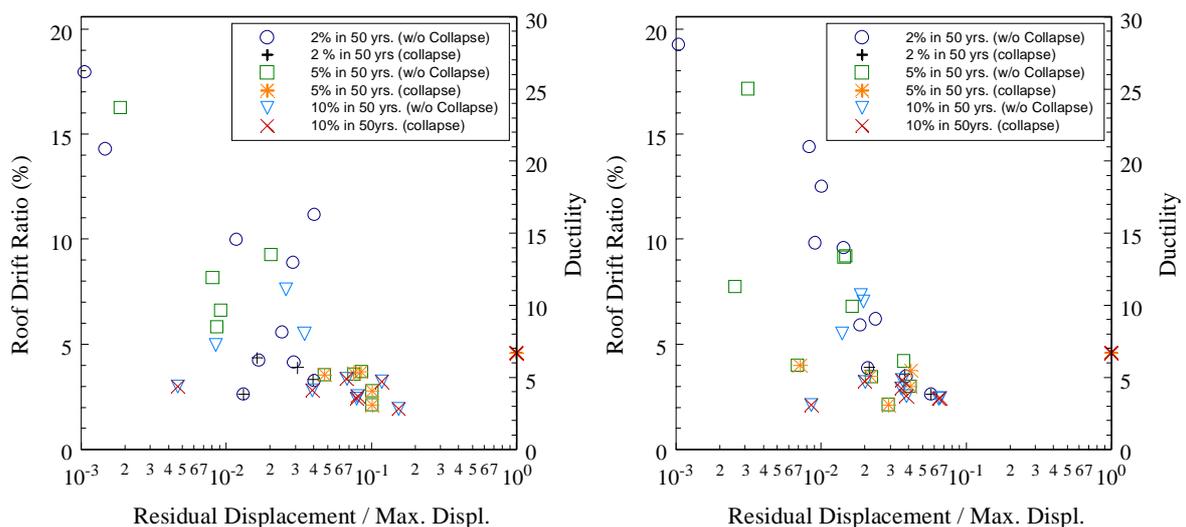


Fig. 6: Performance Matrix of the 12-story RC model building with bilinear (left) and pinching (right) hysteretic behavior.

CONCLUSIONS

A new mathematical form is presented in this study to describe hysteretic loops with post-peak behavior. Unlike other rule-based models, the proposed mathematical form can be readily incorporated into existing in-house and commercial software with no excessive coding efforts. In addition, a preliminary finding of this study is the introduction of performance-based earthquake engineering into the seismic design documents has indicates the necessity of considering post-peak behavior of structural systems into nonlinear dynamic analysis especially at the hazard level of very rare events such as 2% exceedance probability in 50 years.

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