5th IASPEI / IAEE International Symposium
Effects of Surface Geology on Seismic Motion

SELECTION OF GROUND-MOTION PREDICTION EQUATIONS FOR PROBABILISTIC SEISMIC HAZARD ANALYSIS : CASE STUDY OF TAIWAN

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Background

In 2015-June National Center for Research on Earthquake Engineering (**NCREE**), supported by **Tai-Power Company** (to response the request of **NTTF 2.1 Seismic Reevaluation)**, launched NUREG/CR– 6372 researches (SSHAC), in understanding and documenting lessons learned from recent PSHAs conducted at the higher SSHAC Levels.

(follow the experiences of research on **Diablo Canyon Nuclear Power plant)**

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Content

- Introduction
- Selection of candidate GMPEs for PSHA
- Visualization technique for GMPE selection
- Selection of GMPE common form
- Visualization of model space
- Calculate GMPE weighting
- Conclusion

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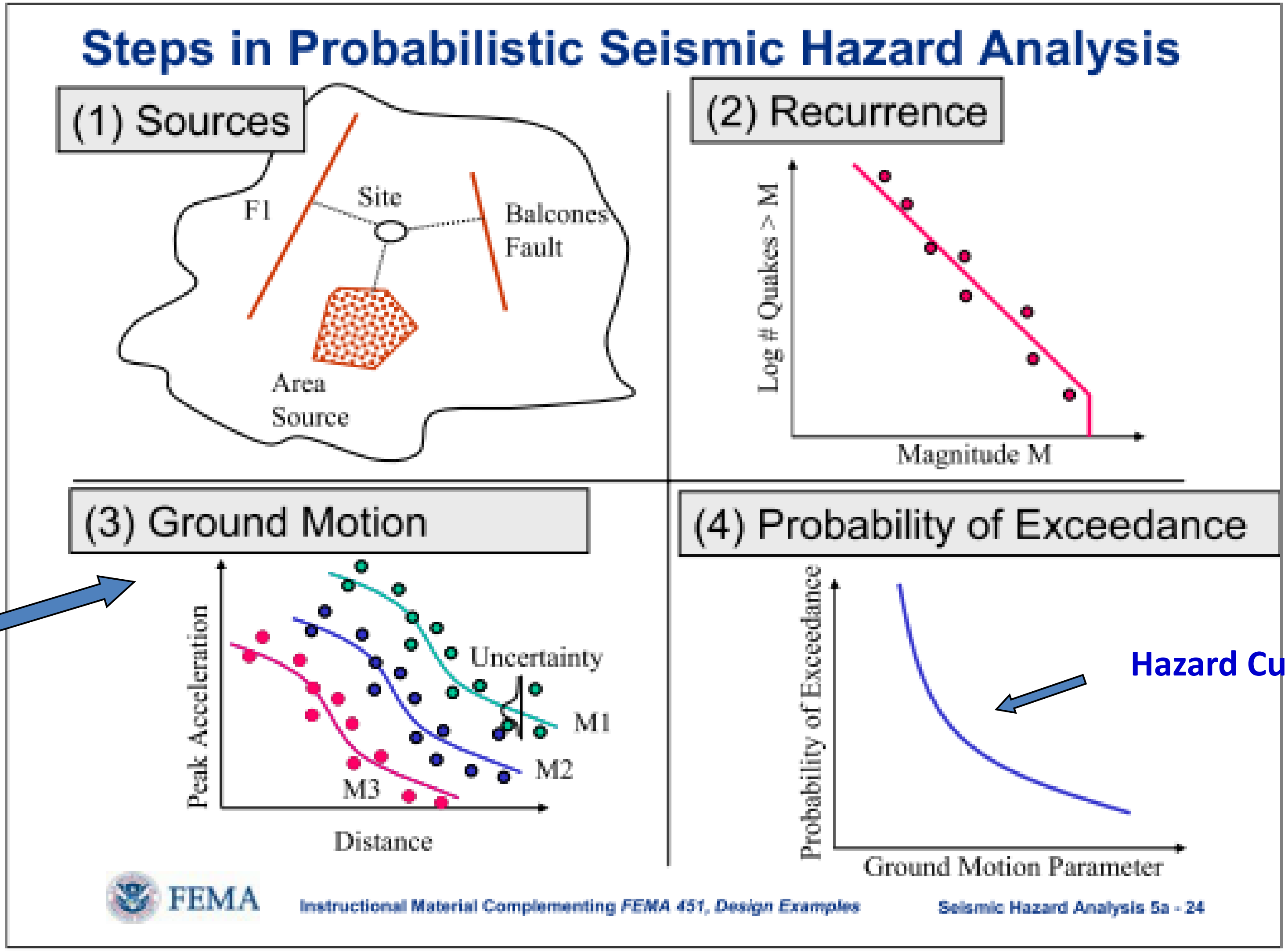
Introduction



Peak Ground Acceleration (PGA) Attenuation

Hazard Curve

Introduction

Selection of appropriate GMPEs for PSHA

1. Need best estimate of GMPE
2. Consider range of alternative models to characterize the uncertainty in the GMPEs

Type of uncertainty in GMPEs

1. **Aleatory uncertainty:**
   expressing random variability of amplitude about a median prediction equation,
   - *can be handled in a PSHA by integrating over the distribution of ground-motion amplitude about the median,*
2. **Espistemic uncertainty:**
   expressing uncertainty concerning the correct value of the median,
   - *can be handled by considering alternative GMPEs in a logic tree format (must capture* ***uncertainties in form & amplitude****),*

Sensitivity analysis of the proposed weights for GMPEs on the seismic hazard.

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General Feature of Candidate GMPEs

GMPE	GMPE Acronym	Regions	Magnitude Interval	Primary distance	Style of faulting	Site effect	Component	Number of records and events
(Abrahamson, N. A., Silva, W. J., and Kamai, R., 2014)	**ASK14**	**Global**	Mw (3.0-8.5)	Rrup (<300km)	SS,NML,REV	Vs30	PGA,PGV,PSA in GMRotI50	15750 and 326
(Boore, D. M., Stewart, J. P ., Seyhan, E., and Atkinson, G. M., 2014)	**BSSA14**	**Global**	Mw (3.0-8.5)	Rjb(<300km)	U,SS,NML,REV	Vs30	PGA,PGV,PSA in GMRotI50	~16000 and ~400
(Campbell, K. W., and Bozorgnia, Y., 2014)	**CB14**	**Global**	Mw (3.3-8.5)	Rrup (<300km)	SS,NML,REV	Vs30	PGA,PGV,PSA in GMRotI50	15521 and 322
(Chiou, B. S-J., and Youngs, R. R., 2014)	**CY14**	**Global**	Mw (3.5-8.5)	Rrup (<300km)	SS,NML,REV	Vs30	PGA,PGV,PSA in GMRotI50	12444 and 300
(Idriss, 2014)	**Id14**	**Global**	Mw (5-8.5)	Rrup (<150km)	SS,NML,REV	Vs30	PGA,PGV,PSA in GMRotI50	7135 and 160
(Akkar, S., Sandikkaya, M. A., and Bommer, J. J., 2014)	**ASB14**	**EU and ME**	Mw (4-7.5)	Rjb (<200km)	SS,NML,REV	Vs30	PGA,PGV,PSA in GM	1041 and 221
(Bindi D., Massa M., Luzi L., Ameri G., Pacor F., Puglia R., and Augliera, P., 2014)	**Bi14**	**EU and ME**	Mw(4-7.6)	Rjb (<300km)	U,SS,NML,REV	Vs30	PGA,PGV,PSA in GM	2126 and 365
(Graizer, V., and Kalkan, E., 2015)	**GK15**	**Global**	Mw(5.0-8.0)	Rrup (<250Km)	SS,NML,REV	Vs30	PGA,PGV,PSA in GM	2583 and 47
Zhao et al. 2016	**Zhao16**	**Japan**	Mw(5.0-7.3)	Rrup(<300km)	FN,SS	Dummy variable	PGA, PSA in GM	6482 and 76 (cr), 47(mum)
Özkan Kale, Sinan Akkar, Anooshiravan Ansari, and Hossein Hamzehloo	**Ka15**	**Turkey and Iran**	Mw(4.0-8.0)	Rjb(<200km)	U,SS,NML,RE V	Vs30	PGA, PGV, PSA in GM	670(Tur),528(Ir)
Lin , P.S et al. 2011	**Lin11**	**Taiwan**	Mw(5.0-7.6)	Rrup(<240km)	-	no	PGA,PSA in GM	5268 and 52
(Cauzzi, C., Faccioli, E., Vanini, M., and Bianchini, A., 2014)	**Ca14**	Global	Mw(4.5-7.9)	Rrup (<150km)	U,SS,NML,REV	Vs30	PGA,PGV,PSA in GM	1880 and 98

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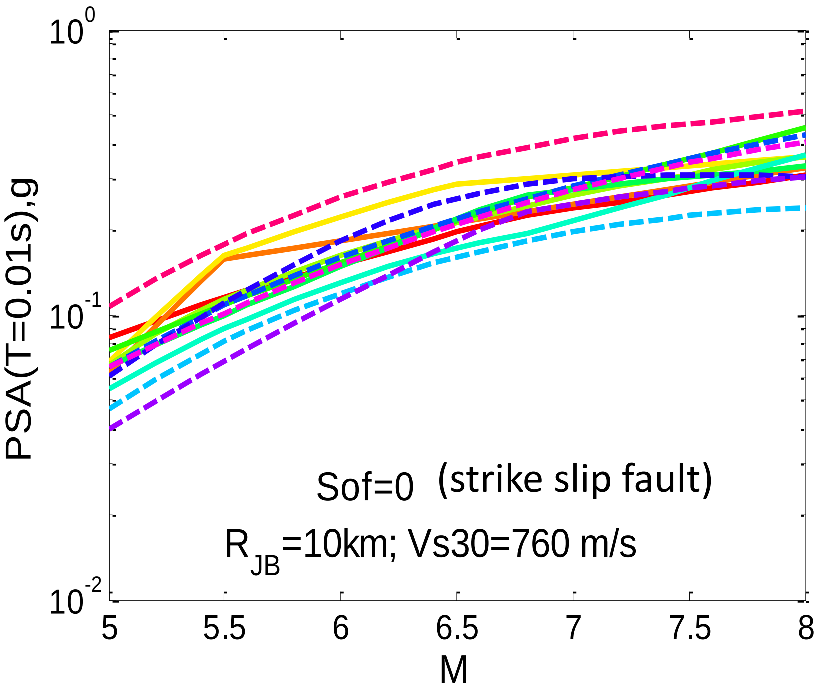
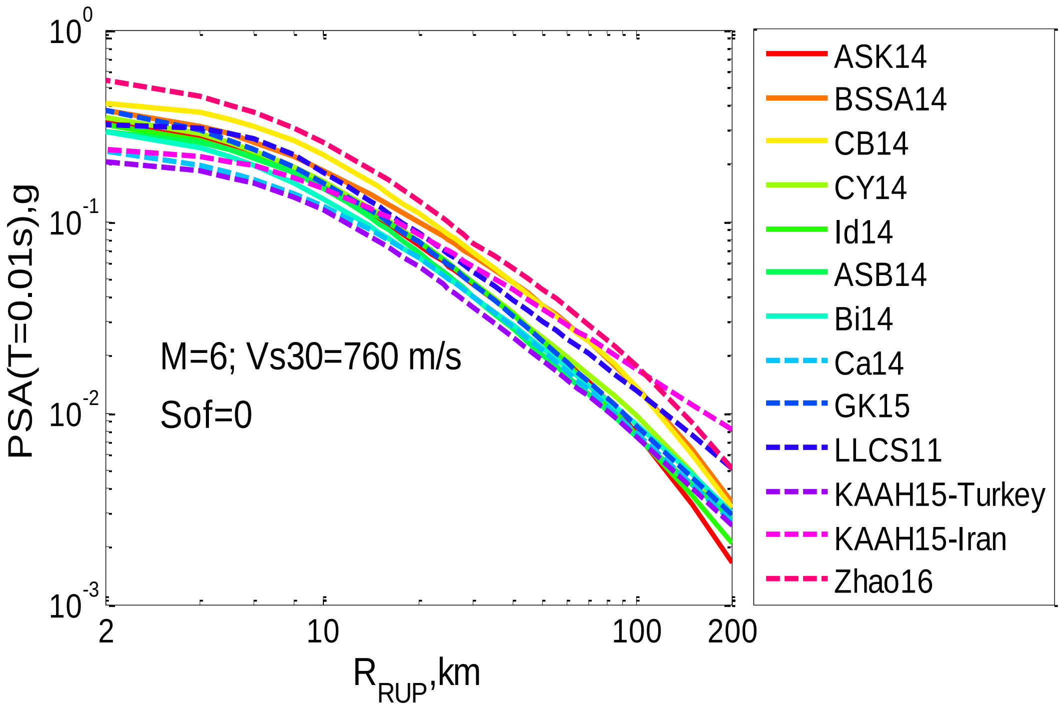
Selection of candidate GMPE

- Selected GMPEs: **ASK14, BSSA14, CB14, CY14, Id14, GK15, ASB14, Bi14, Ca14 Lin11, KAAH15-Turkey, KAAH15-Iran, Zhao16** (total of 13 models)





Use of multiple models with alternative functional forms is required to properly capture uncertainties in **forms** as well as in **amplitude**.

Develop Mix Model from scenatios of candidate GMPEs

- **Selected GMPEs**: **ASK14, BSSA14, CB14, CY14, Id14, ASB14, Bi14, Ca14, GK15, Lin11, KAAH15-Turkey, KAAH15-Iran, Zhao16**

  (total of 13 models)

$$Mix = \sum_{i=1}^{N} w_i GMPE_i\left(M, R, \theta\right)$$

- The scenarios for generate synthetic data:
  - M = 5.0, 5.2, 5.4, 5.5, 5.6, 5.8, 6.0, 6.2, 6.4, 6.5, 6.6, 6.8, 7.0, 7.2, 7.4, 7.5, 7.6, 7.8, 8.0 **for strike slip and reserve faulting**.
  - M = 5.0, 5.2, 5.4, 5.5, 5.6, 5.8, 6.0, 6.2, 6.4, 6.5, 6.6, 6.8, 7.0 **for normal faulting.**
  - Rx=-200,-150,-100,-85,-70,-65,-60,-55,-50,-45,-40,-35,-30,-28,-26,-24,-22,-20,-18,-16,-15,-14,-12,-10,-8,-6,-5,-4,-2. (foot wall)
  - **From fault geometry, Rrup, and Rjb can be calculated**.
  - **Vs30 = 760 m/s.**
  - **Dip =90^o for strike slip, and dip = 45^o for normal and reverse faulting events**.
  - Other parameters are set to default (Ztor, W,...)

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Reference to the set of 13GMPEs

✓ **Add the reference to the set of 13GMPEs:**

- ❑ Mix Model (average of all models) :

$$Mix = \sum_{i=1}^{N} w_i GMPE_i\left(M, R, \theta\right)$$

- ❑ Up-Down Scaled models :

$$Mix + \log\alpha,\ \text{with } \alpha = 0.67,\ 0.8,\ 1.25,\ 1.5\ \square\ \text{S--, S-,S+,S++}$$

- ❑ Magnitude Scaled models :

$$Mix + \beta\left(M - 6.5\right),\ \text{with } \beta = \text{-0.4, -0.2, 0.2, 0.4}\ \square\ \text{M--, M-, M+, M++}$$

- ❑ Distance Scaled models:

$$Mix + \gamma\left(R - 70\right),\ \text{with } \gamma = \text{-0.01, -0.005, 0.005, 0.01}\ \square\ \text{R--, R-, R+, R++}$$

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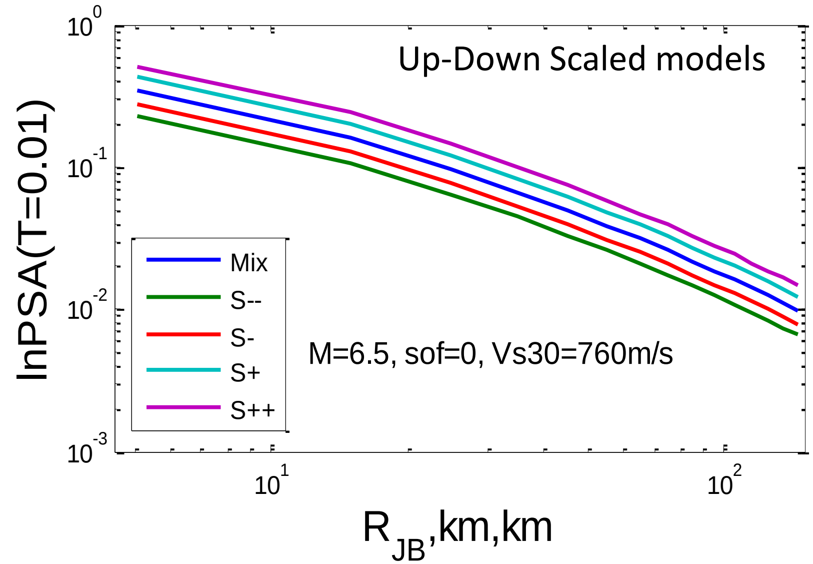
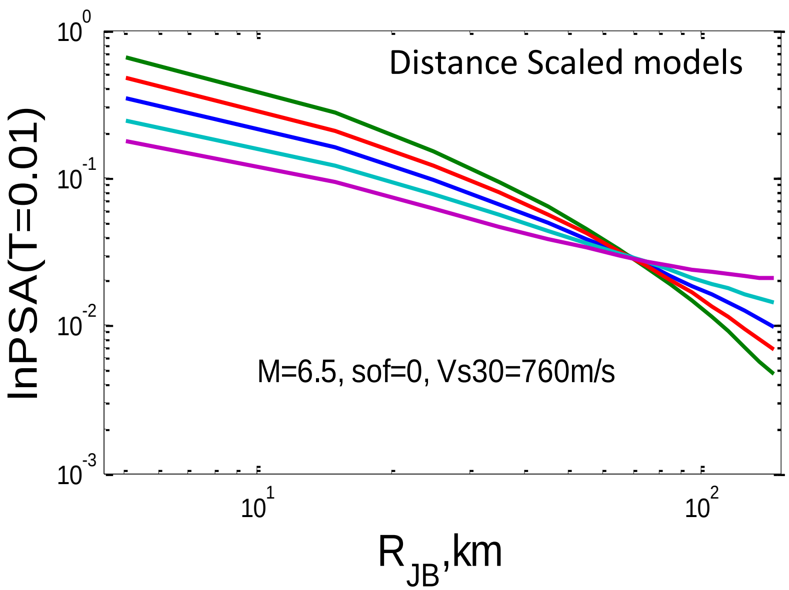
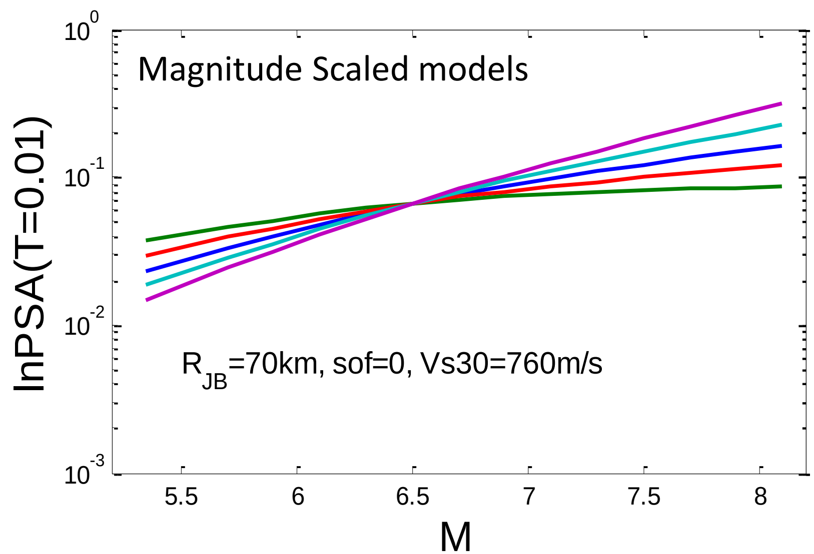
Reference to the set of 13GMPEs







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Generate Sammon Map

(13 Candidate GMPEs) + (13 Reference models: mix & scale models)

$$GMPE_i = f(M, R) \quad \begin{cases} M = 5.0, 5.2, \ldots, 7.8, 8.0 \text{ (for SS \& NF)} \\ \text{Rx=-200,-150, ..., -4,-2. (foot wall)} \end{cases}$$

The simplest technique **for dimensionality reduction** is a straightforward **linear projection**, for example, as in PCA — principal component analysis. (PCA simply maximizes variance)

Non-linear projections may therefore be desirable when analyzing such data.

Sammon Mapping:

To minimize the differences between corresponding inter-point distances in the dimension space

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- Combined the embedded **GMPEs** and the **Mix Model & Scaled Models** into the [*X*] matrix considered to be N-dimensional space

$$[\boldsymbol{\mu}_{GMPE}]_{13xN} = \begin{bmatrix} ASK14(M,R) \\ BSSA14(M,R) \\ \vdots \\ CY14(M,R) \\ Lin11(M,R) \end{bmatrix}_{13xN}$$

$$[\boldsymbol{\mu}_{scales}]_{13xN} = \begin{bmatrix} \langle Mix \rangle_{1xN} \\ [S]_{4xN} \\ [M]_{4xN} \\ [R]_{4xN} \end{bmatrix}_{13xN}$$

$$[X]_{26xN} = \begin{bmatrix} [\boldsymbol{\mu}_{GMPE}]_{13xN} \\ [\boldsymbol{\mu}_{Scaled}]_{13xN} \end{bmatrix}$$

$$[X]_{2D-pca} = \left(\frac{XX^T}{N-1} \right) \cdot \left[\Phi_{pca} \left(\sim \lambda_1, \lambda_2 \right) \right]$$

Construct the initial PCA-based map

$$[X]_{2D-PCA} = \begin{Bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_{26} & y_{26} \end{Bmatrix}$$

2D visualization (preliminary)

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Visualization technique-GMPEs Calculation

- To construct the $\left[X_{Sammon-map}\right]_{26x2}$

Using $[X]_{26xN} = \begin{bmatrix} [\boldsymbol{\mu}_{GMPE}]_{13xN} \\ [\boldsymbol{\mu}_{Scaled}]_{13xN} \end{bmatrix}$

Calculate inter-point distance

and define $\varepsilon_{ij} = \sqrt{\left(\sum_{k=1}^{N}([X_{ik}]-[X_{jk}])^2\right)/N}$

to construct $[\Delta_{[X]}]_{26x26} = \begin{bmatrix} 0 & \varepsilon_{X1-2} & \cdots & \varepsilon_{X1-26} \\ \varepsilon_{X2-1} & 0 & \cdots & \varepsilon_{X2-26} \\ \cdots & \cdots & \ddots & \vdots \\ \varepsilon_{X26-1} & \varepsilon_{X26-2} & \cdots & 0 \end{bmatrix}_{26x26}$

Define $\delta_{Xi,j} = \sqrt{\sum_{p=1}^{2}(x_{Xi,p} - x_{Xj,p})^2}$

$$[\Delta_{PCA-Map}]_{26x26} = \begin{bmatrix} 0 & \delta_{X1,2} & \cdots & \delta_{X1,26} \\ \delta_{X2,1} & 0 & \cdots & \delta_{X2,26} \\ \cdots & \cdots & \ddots & \vdots \\ \delta_{X26,1} & \delta_{X26,2} & \cdots & 0 \end{bmatrix}_{26x26}$$

$\Rightarrow$ ***Min.*** $$E = \frac{1}{\sum_{i<j}\varepsilon_{ij}} \sum_{i<j} \frac{(\varepsilon_{ij} - \delta_{ij}^{map})^2}{\varepsilon_{ij}}$$

Determine : Δ_{PCA}^{Map} to construct the Sammon map $\left[X_{Sammon-map}\right]_{26x2}$

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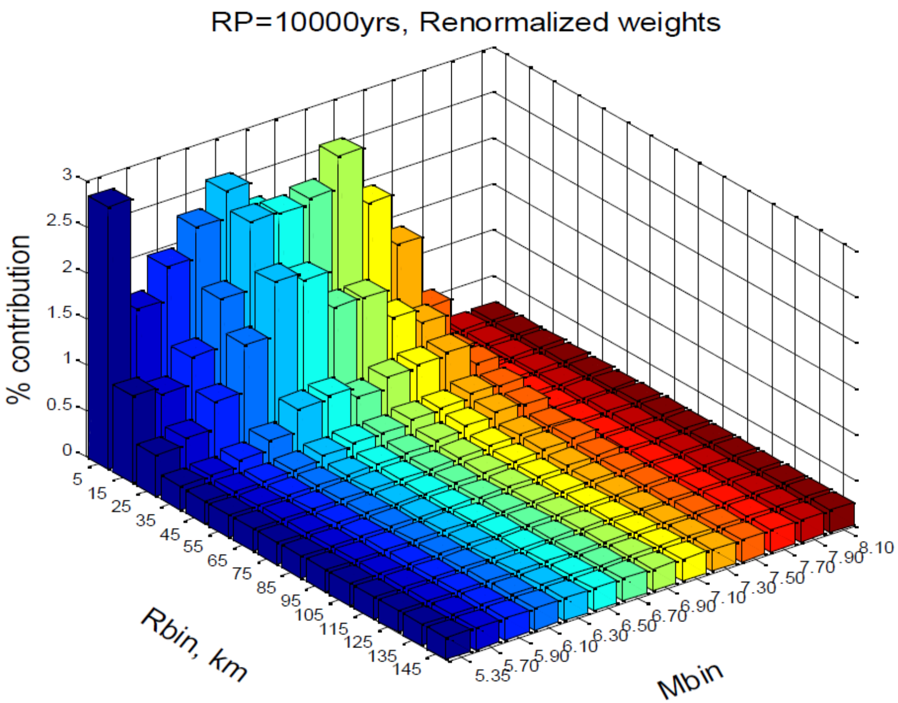
Visualization technique-GMPEs Calculation

$$\varepsilon_{ij} = \sqrt{\left(\sum_{k=1}^{N}([GMPE_{ik}]-[GMPE_{jk}])^2\right)/N}$$

$$[\Delta_{\{X\}}]_{26x26} = \begin{bmatrix} 0 & \varepsilon_{X1-2} & \cdots & \varepsilon_{X1-26} \\ \varepsilon_{X2-1} & 0 & \cdots & \varepsilon_{X2-26} \\ \cdots & \cdots & \ddots & \vdots \\ \varepsilon_{X26-1} & \varepsilon_{X26-2} & \cdots & 0 \end{bmatrix}_{26x26}$$

$$\overline{\Delta}_{ij}^{GMPE} = \sqrt{w_k \sum_{k}^{N}\left(GMPE_{ik} - GMPE_{jk}\right)^2}$$

$$w_k = 0.5\left(DEAGG\left(M_k, R_k\right) + \frac{1}{NS}\right)$$



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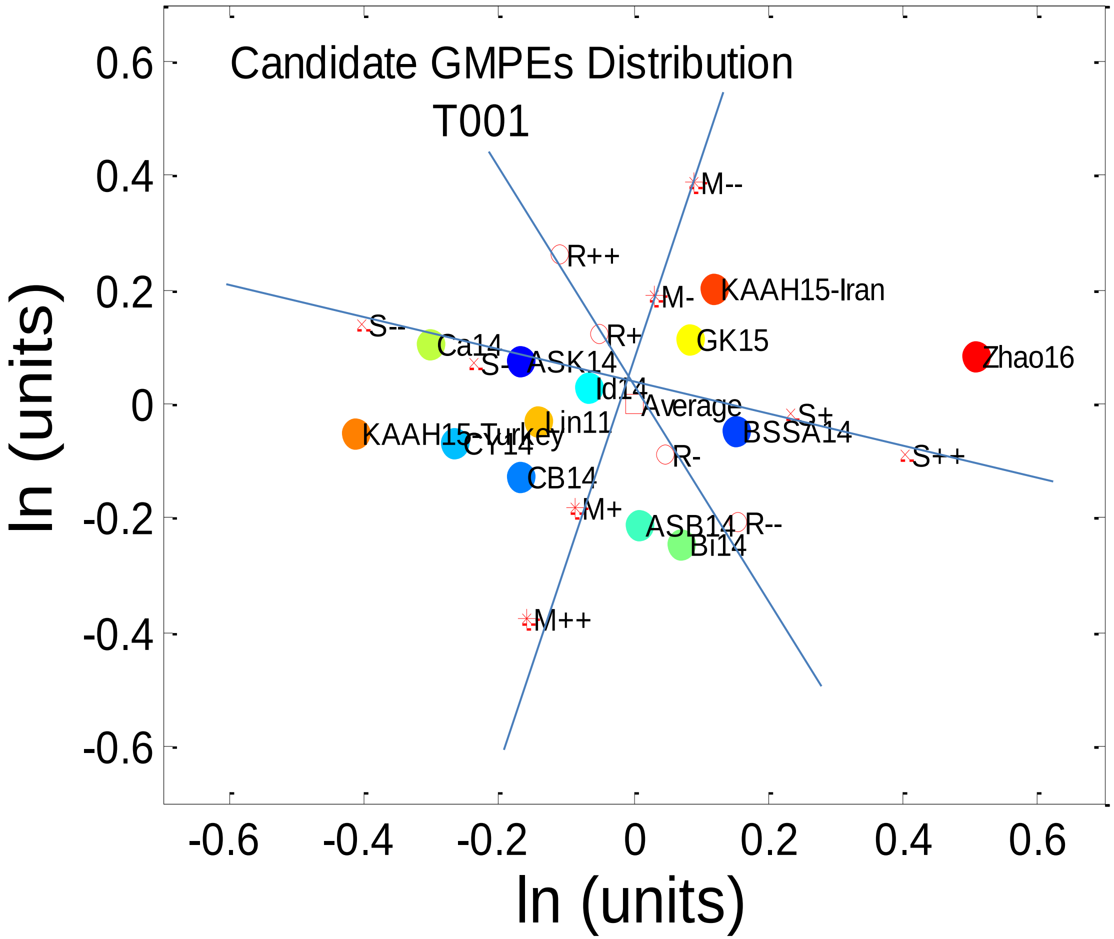
- Visualization of models in 2-Ddimension

$$[X]_{Sammon-Map} = \begin{Bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_{26} & y_{26} \end{Bmatrix}$$



The considered GMPE models are not adequate to fully capture the range of epistemic uncertainty because of the existing gaps among models.

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Selection of candidate GMPE common form

Model A - based on rupture distance, R_{RUP}

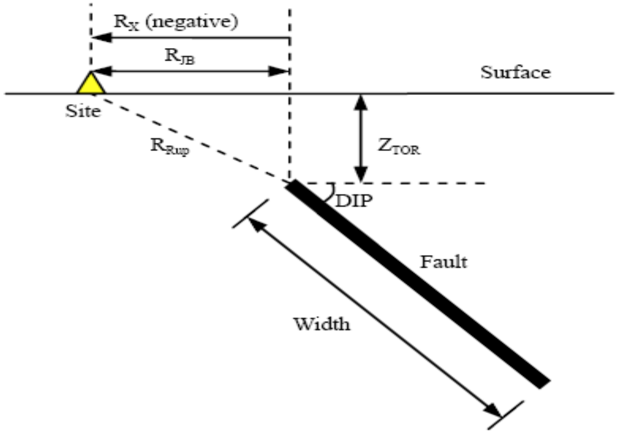
$$SA_{Rrup-based} = \theta_1 - \theta_8^2 R_{rup} + \theta_9^2 Z_{tor} - \exp(\theta_{11}) F_{NML} + \exp(\theta_{10}) F_{REV} + \left(\theta_5 + \theta_6 (M-5)\right) \ln\left(\sqrt{R_{rup}^2 + \theta_7^2}\right)$$

$$+ \begin{cases} \theta_2 M & \text{for } M < 5.5 \\ \theta_2 M + \theta_3 (M\text{-}5.5) & \text{for } 5.5 \le M \le 6.5 \\ \theta_2 M + \theta_3 (M\text{-}5.5) + \theta_4 (M - 6.5) & \text{for } M > 6.5 \end{cases}$$

Model B - based on rupture distance, R_{JB}

$$SA_{Rjb-based} = \theta_1 - \theta_8^2 R_{jb} - \exp(\theta_{11}) F_{NML} + \exp(\theta_{10}) F_{REV} + \left(\theta_5 + \theta_6 (M-5)\right) \ln\left(\sqrt{R_{jb}^2 + \theta_7^2}\right)$$

$$+ \begin{cases} \theta_2 M & \text{for } M < 5.5 \\ \theta_2 M + \theta_3 (M\text{-}5.5) & \text{for } 5.5 \le M \le 6.5 \\ \theta_2 M + \theta_3 (M\text{-}5.5) + \theta_4 (M - 6.5) & \text{for } M > 6.5 \end{cases}$$



(Behave differently on the hanging wall side)

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Select common form to fit the synthetic data

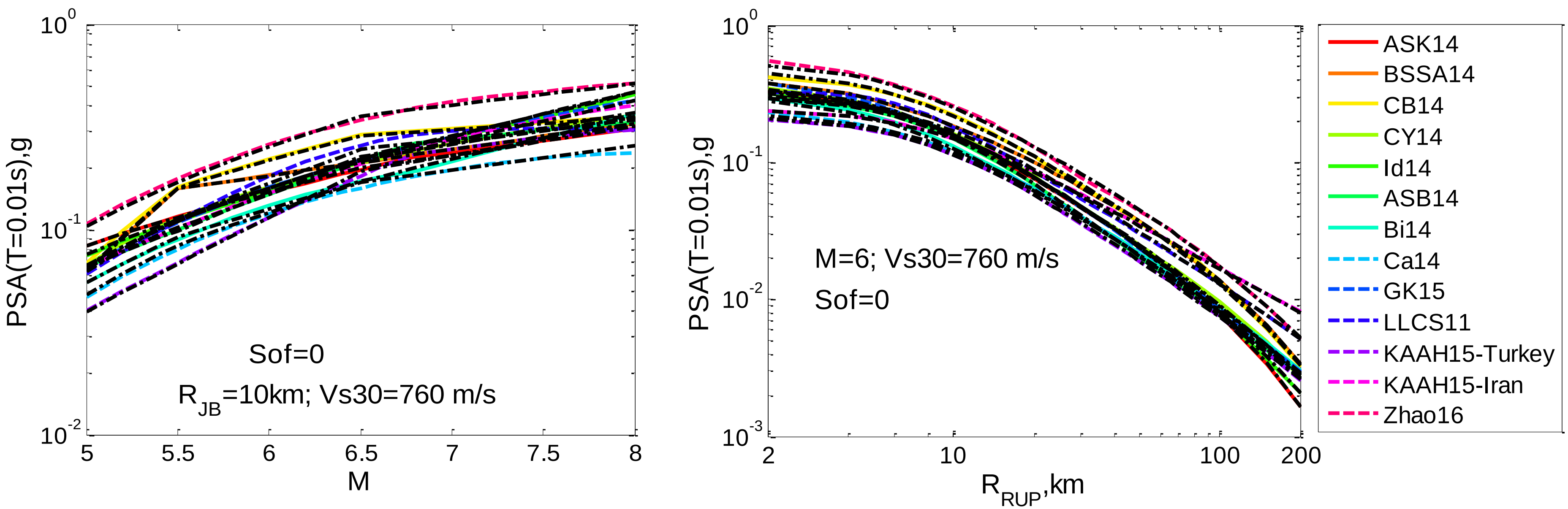
GMPE Distribution

Develop common funtional form for each candidate GMPE

$$\mu(\ln y) = f\left(M, R, \theta, \ldots\right)$$

For each GMPE *i*, estimate the set of coefficients θ_i (using senthetic data)

Comparison between the common form GMPE with respect to the candidate GMPE



(Sof=0) : strike slip fault

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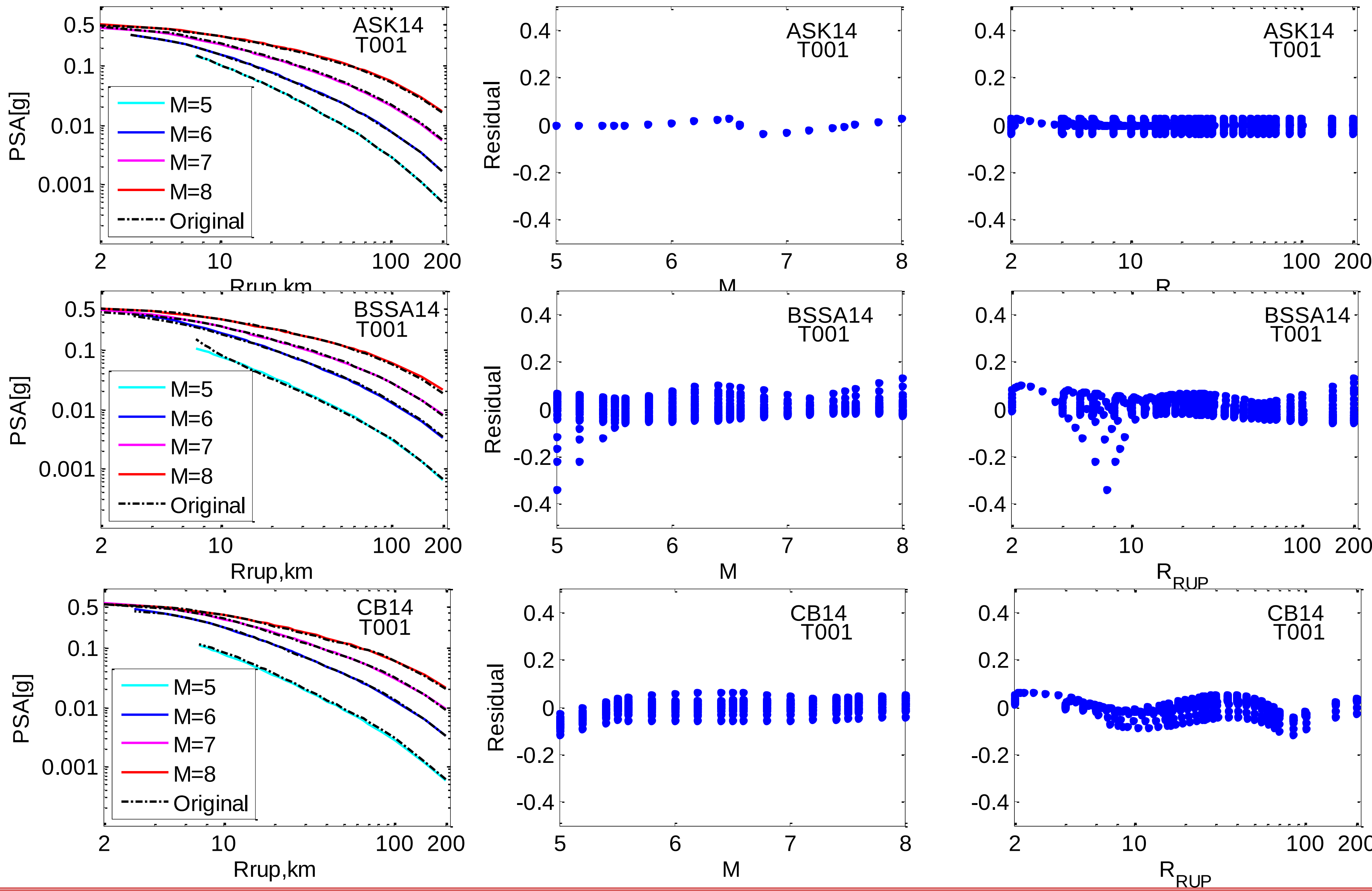
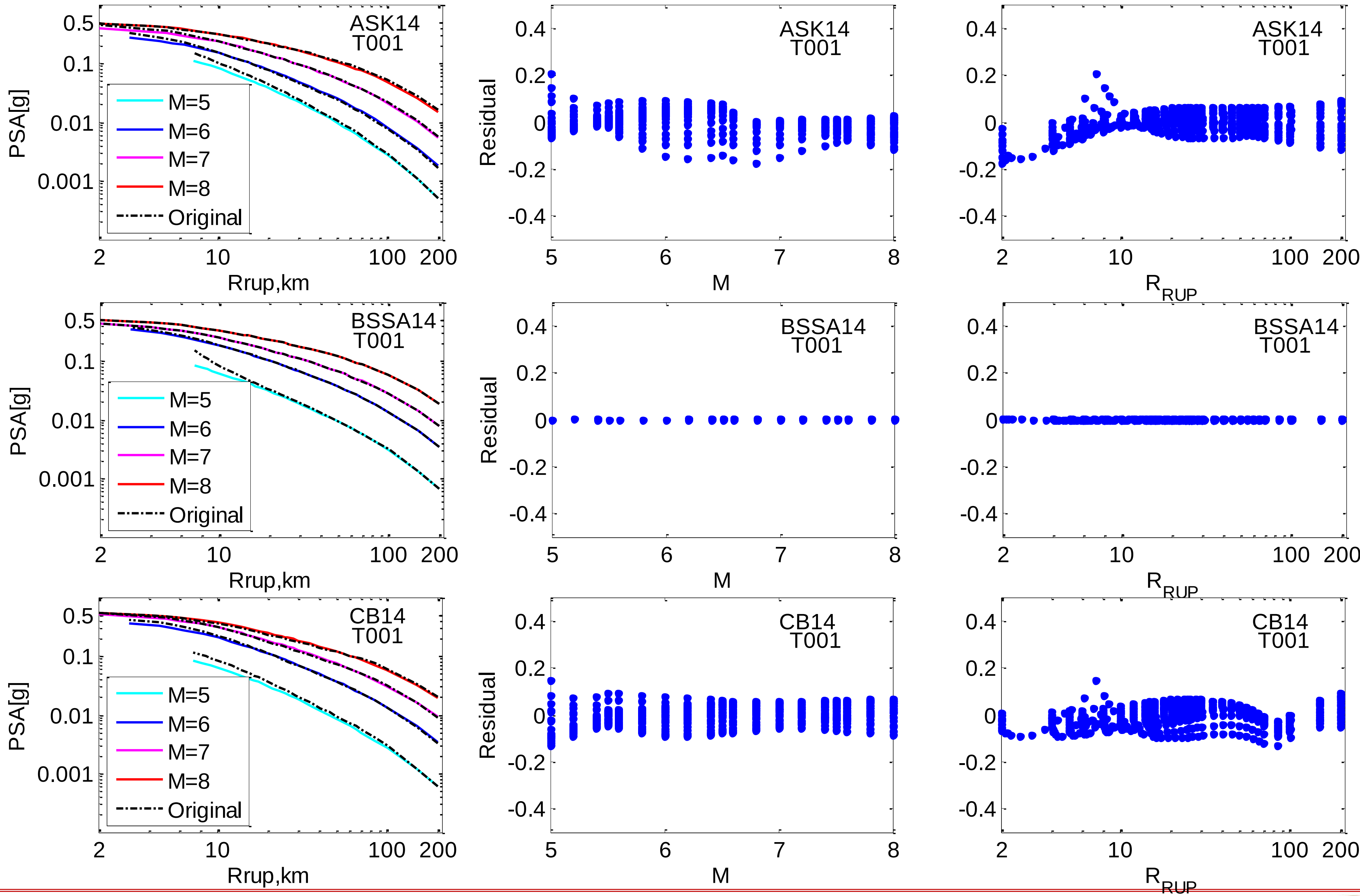
Example : Fit common form to synthetic data. R_{RUP}-based



Example: Fit common form to synthetic data. R_{JB}-based



Select common form to fit the synthetic data

Develop common funtional form for each candidate GMPE

$$\mu(\ln y) = f(M, R, \theta, \ldots)$$

For each GMPE i, estimate the set of coefficients θ_i (using senthetic data)

In order to capture the correlation between the different coefficients θ, the common form is also fitted to the interpolated ground motions from the candidate GMPEs

$$< \theta_1, \cdots, \theta_{10}, \theta_{11} >_{GMPE-1}$$
$$< \theta_1, \cdots, \theta_{10}, \theta_{11} >_{GMPE-2}$$
$$\cdots\cdots$$
$$< \theta_1, \cdots, \theta_{10}, \theta_{11} >_{GMPE-13}$$
$$\vdots$$

$$Interp(\ln SA(M,R,Vs30)) = w_a \ln(SA_i) + w_b \ln(SA_j) \text{ with } i \neq j$$

$$\{w_a, w_b\} = \left\{\frac{1}{3}, \frac{2}{3}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{2}{3}, \frac{1}{3}\right\}$$

Calculate mean μ_θ and covariance Σ_θ

Given μ_θ and Σ_{θ}, sample new sets of coefficients $\{\theta\}$ and thus generate new models

(Assumed multivariate normal distribution)

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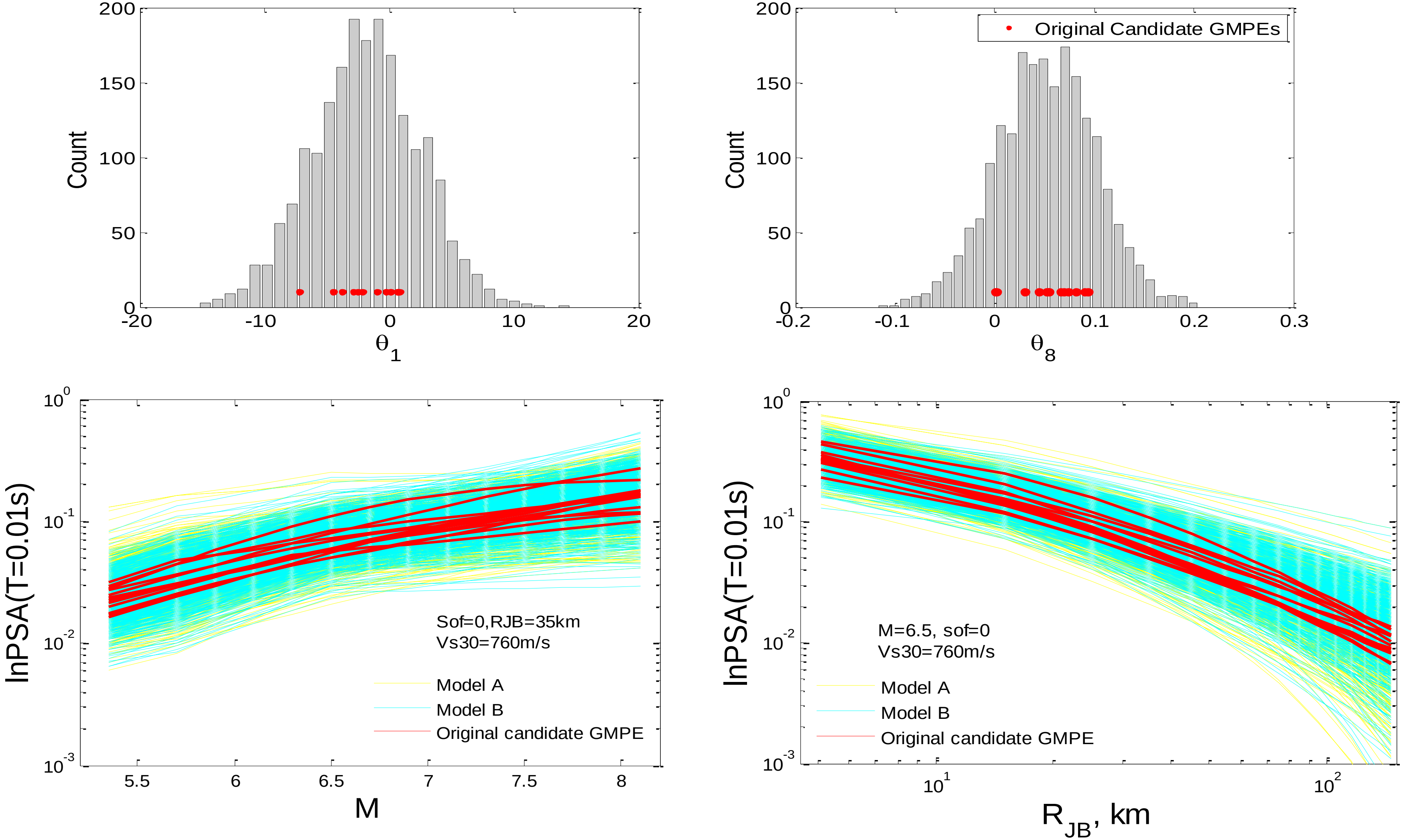
Distribution of GMPE coefficient

Calculate μ_θ and Σ_θ (from the fitted sets of coefficients)



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GMPE Distribution

- Visualization of models in 2D
  - M = 4.75, 5.25,...,7.75
  - RJB = 1.25, 1.5, 3.75, ...,87.5
  - Vs30=760m/s
  - Sof = 0, 1
  - T=.001s

Convex hull of candidate GMPE with plus/minus 2σ_{AY14}

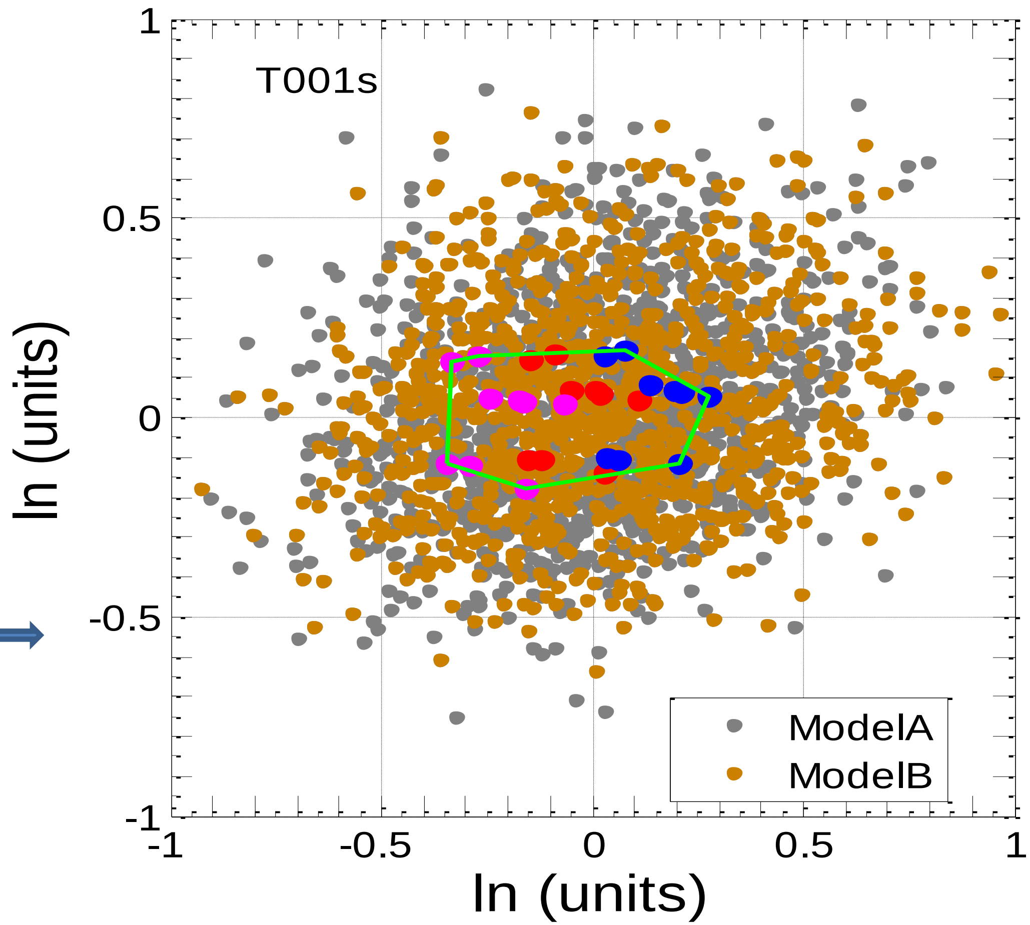
Zhao16 and KAAH-Turkey were not included

Original Candidate GMPEs
Original Candidate GMPEs+2σ
Original Candidate GMPEs-2σ

$$\ln PSA_{ij} = f_i\left(M_j, R_j, \ldots\right) + \alpha\sigma_{AY14}\left(M_j, R_j, F\right)$$

with α= {-2, 0, 2}

Model A: R_{RUP}-based
Model B: R_{JB}-based



Capture the range of epistemic uncertainty.

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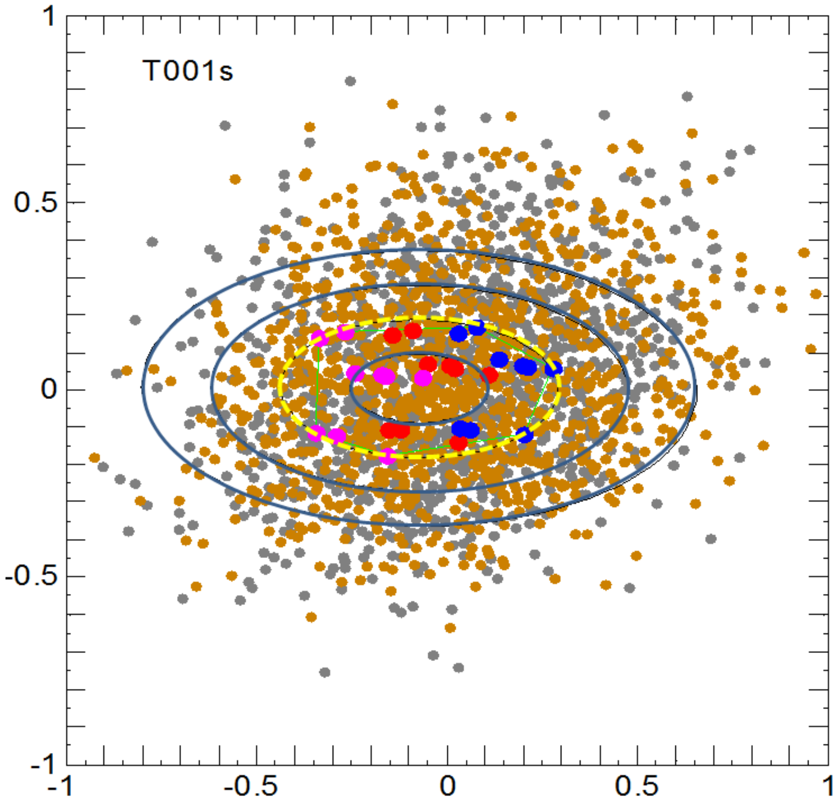
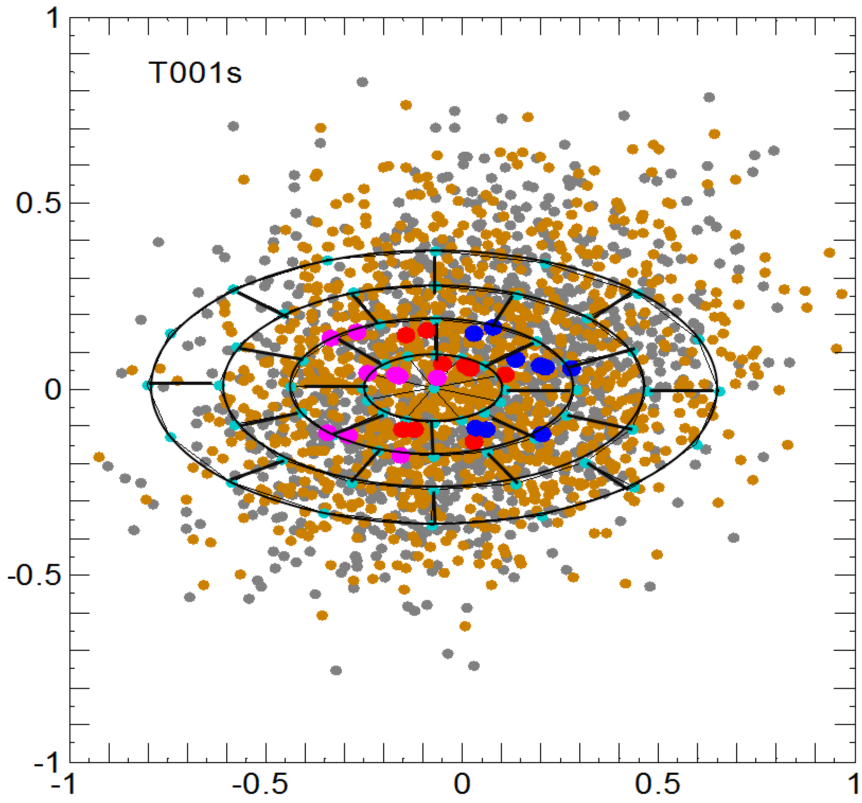
Fitting ellipse to convex hull.
Scale up and down by a factor 2, 1.5 and 0.5

Split region covered by four ellipese into many subregion.





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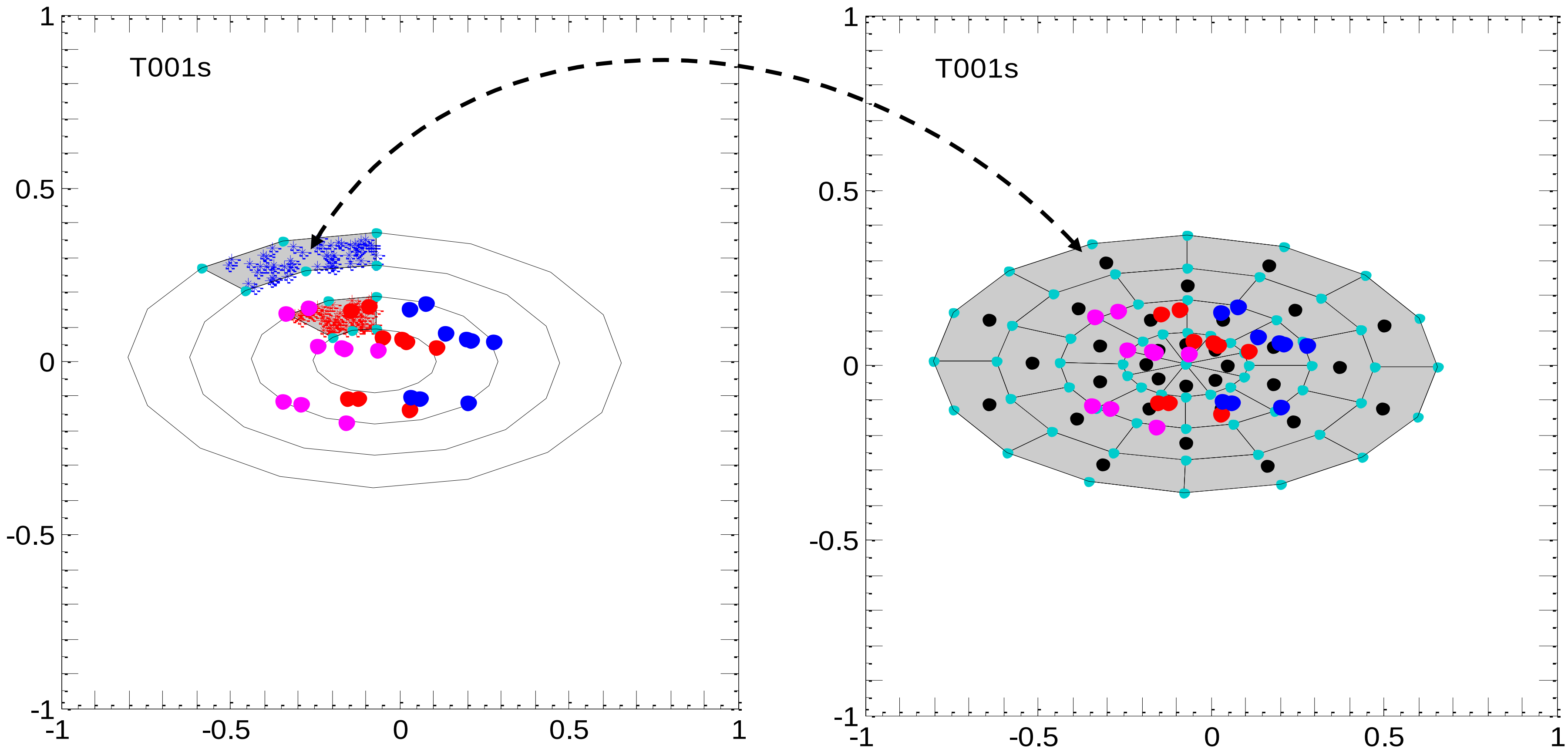
Selection of models – Approach 1



- **For each subregion, common forms are selected**
- **Select representative model for each subregion.**

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Visualization of model space: Residual analysis

- Database. (about 900 records from Taiwan area)
  - Selected data that is relevant for the application: M≥5, Rrup≤100km, and 700 ≤Vs30≤ 800m/s.

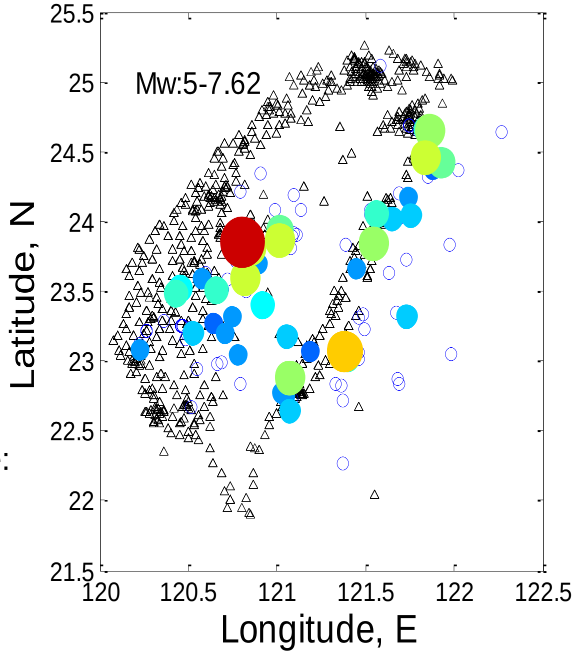
  → Corrected to Vs30 = 760m/s.

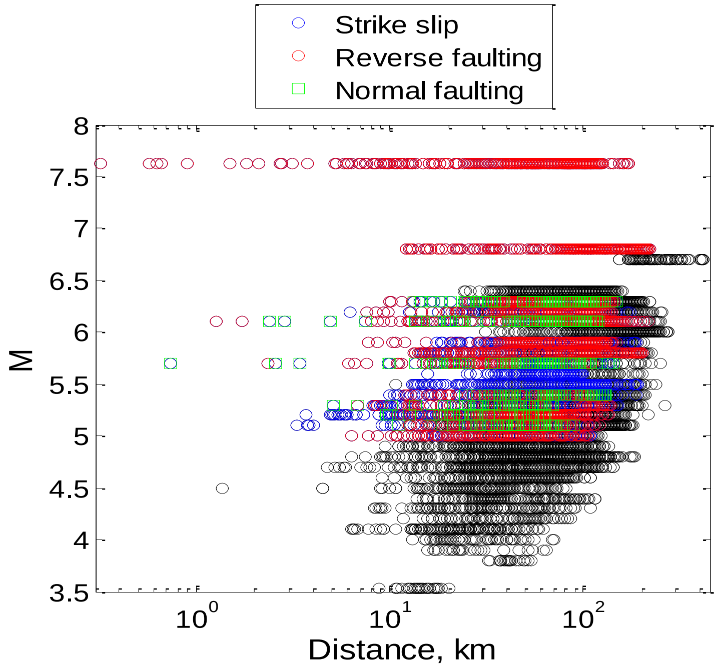
- For each of 2000 sample models.
  - Residual are calculated.
    - These are split into between-event residual and within event residuals, δB *and* δw_{ij}
  - The log-likelihood is calculated using the eq.7 of Abrahamson and Youngs 1992

$$E_{ij} = \eta_i + \varepsilon_{ij} \sim \delta B_e + \delta W_{es}$$

$$\sigma^2 = \tau^2 + \phi^2$$

Shallow Crustal Earthquake:
M:5-7.62
Vs30>150m/s.





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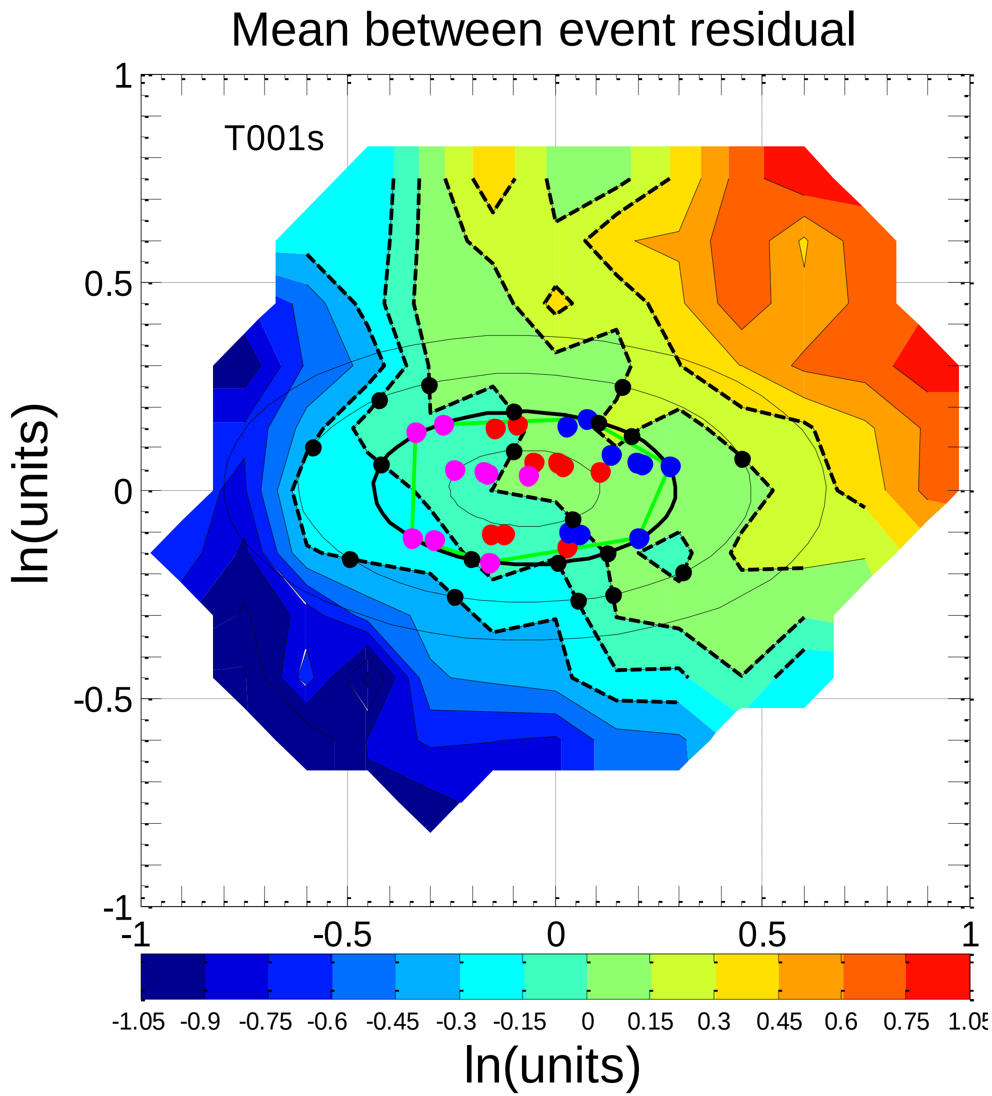
GMPE Distribution

- Scenarios
  - M = 4.75, 5.25,...,7.75
  - RJB = 1.25, 1.5, 3.75, ...,87.5
  - Vs30=760m/s
  - Sof = 0, 1
  - T=0.001s
- Data corrected to Vs30=760m/s
- Model evaluation.
  - **Contour using mean between event residual + Candidate GMPEs**

Overlay the contour map of mean between residual w.r.t. the fitting ellipse



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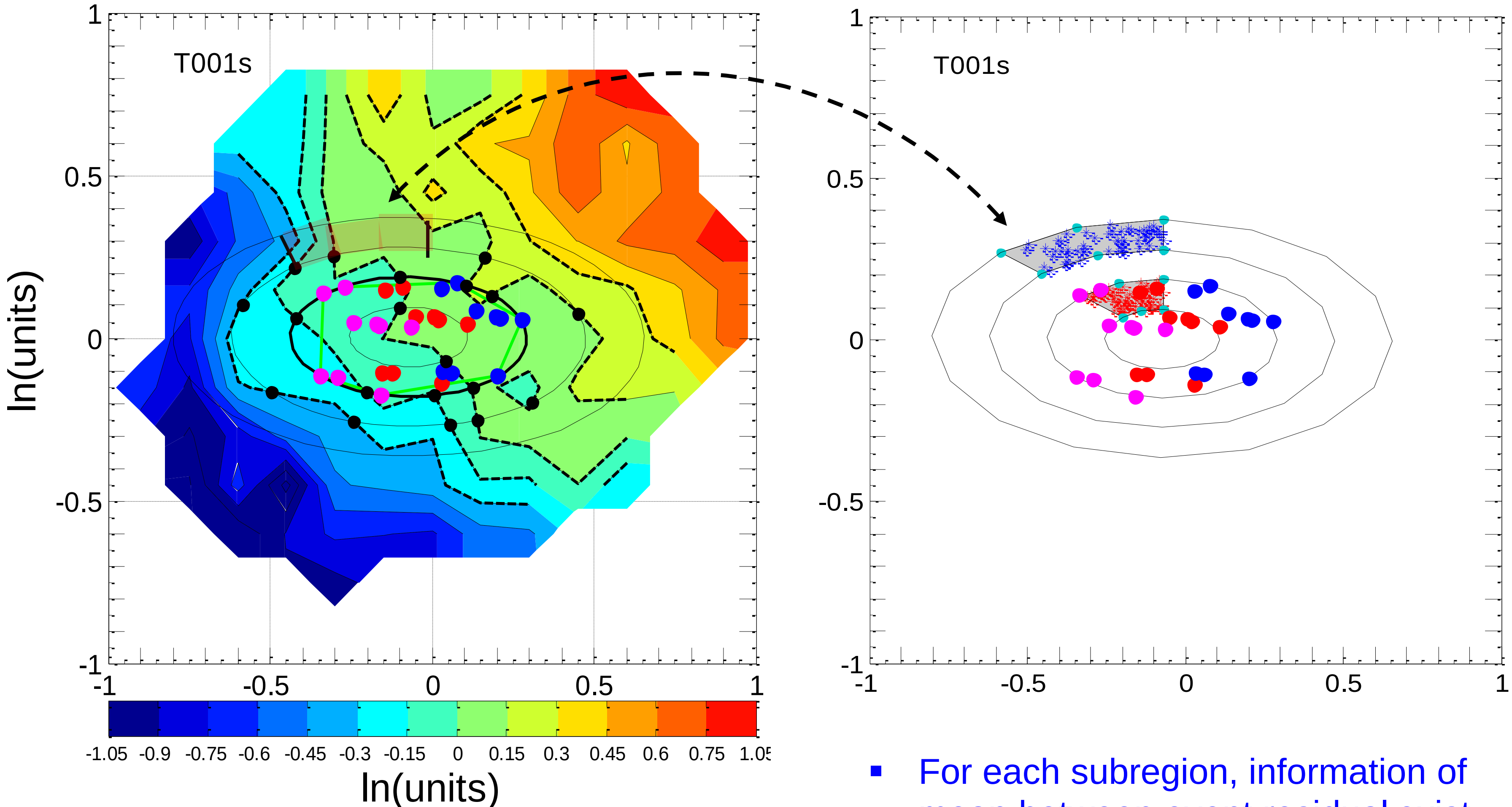
Selection of models – Approach 1

Identify the value of mean between event residual of each form from each split region.



- For each subregion, information of mean between event residual exist.

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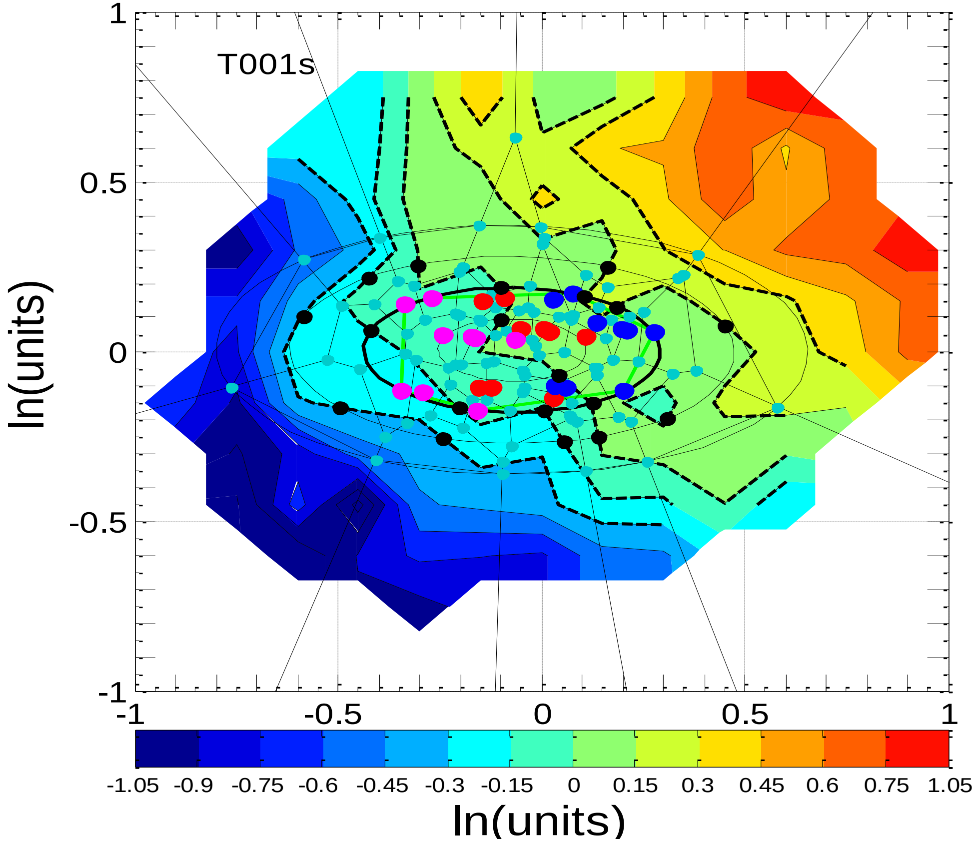
Selection of models – Approach 2

- Discretizing the map using the Voronoi-Diagram based on a set of points (includes median GMPEs, median$\pm 2\sigma_{AY14}$, and the points results from the intersection between contours and ellipses).
- This decomposition has the property that an arbitrary point P within the region R{i} is closer to point i than any other points.

- **Discretizing the map using the Voronoi-Diagram.**



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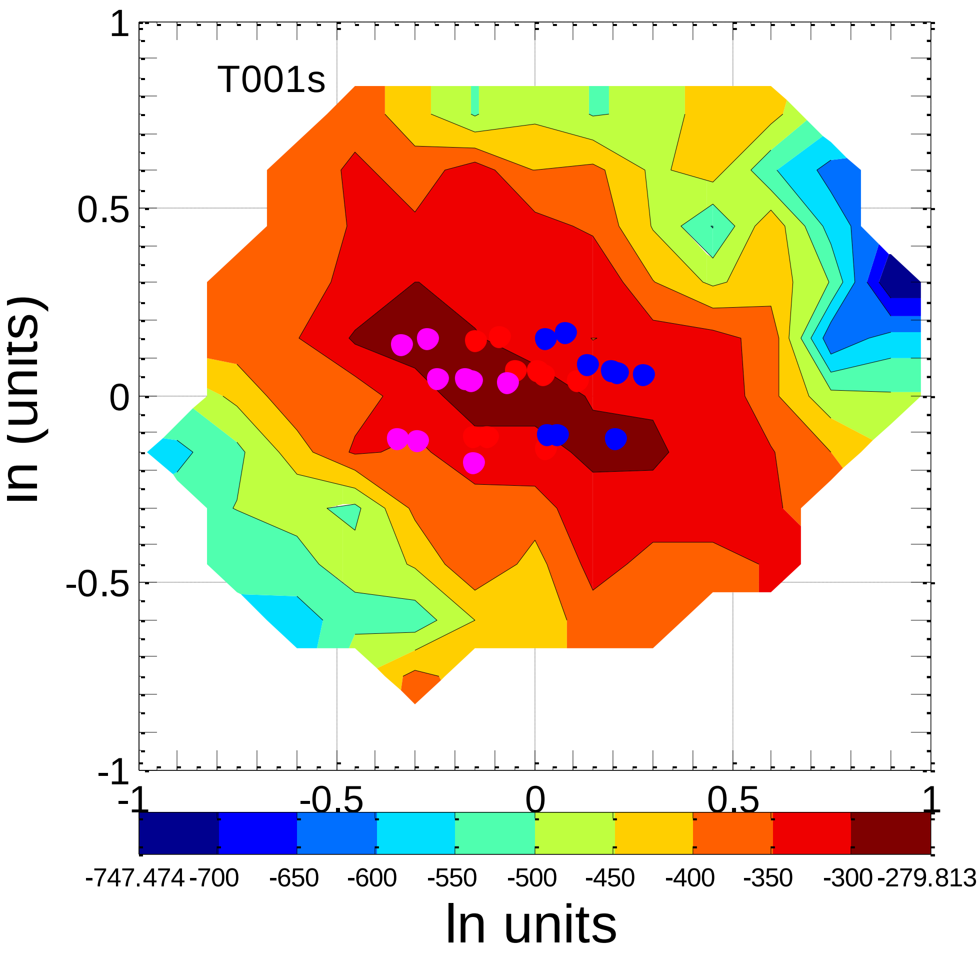
GMPE Distribution

- Scenarios
  - M = 4.75, 5.25,...,7.75
  - RJB = 1.25, 1.5, 3.75, ...,87.5
  - Vs30=760m/s
  - Sof = 0, 1
  - T=.001s
- Data corrected to Vs30=760m/s
- Model evaluation.
  - **Contour based on Log-likelihood value of each model**

- Log-likelihood



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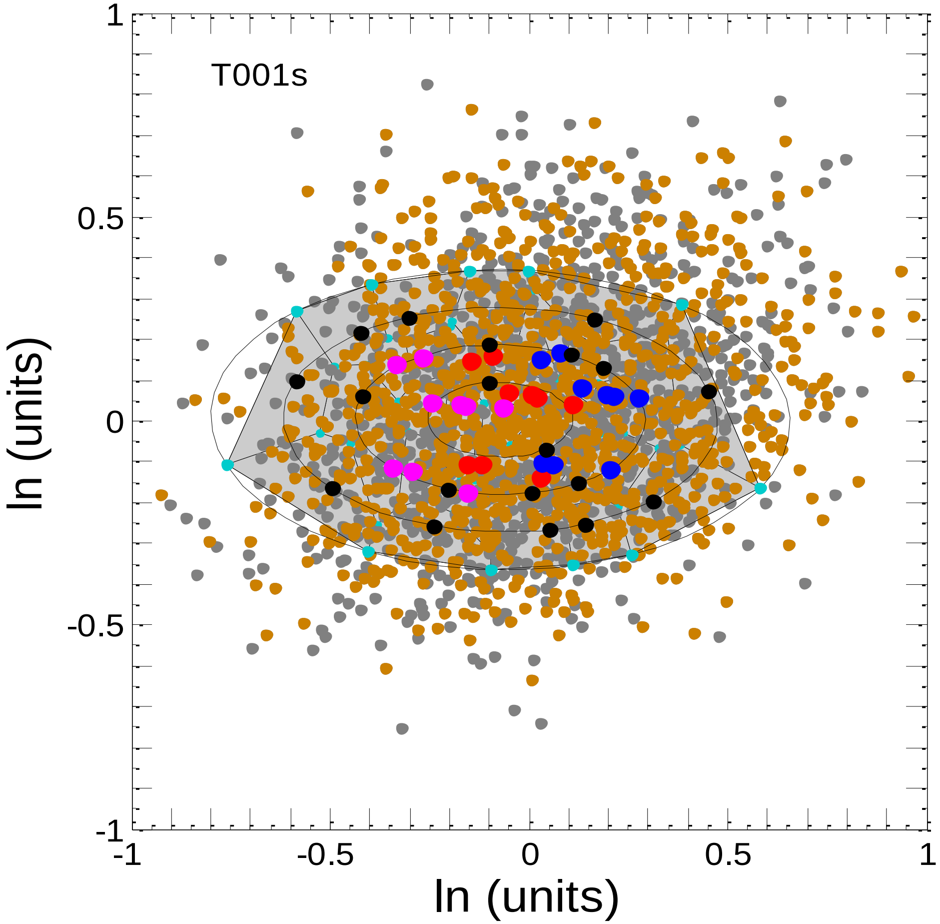
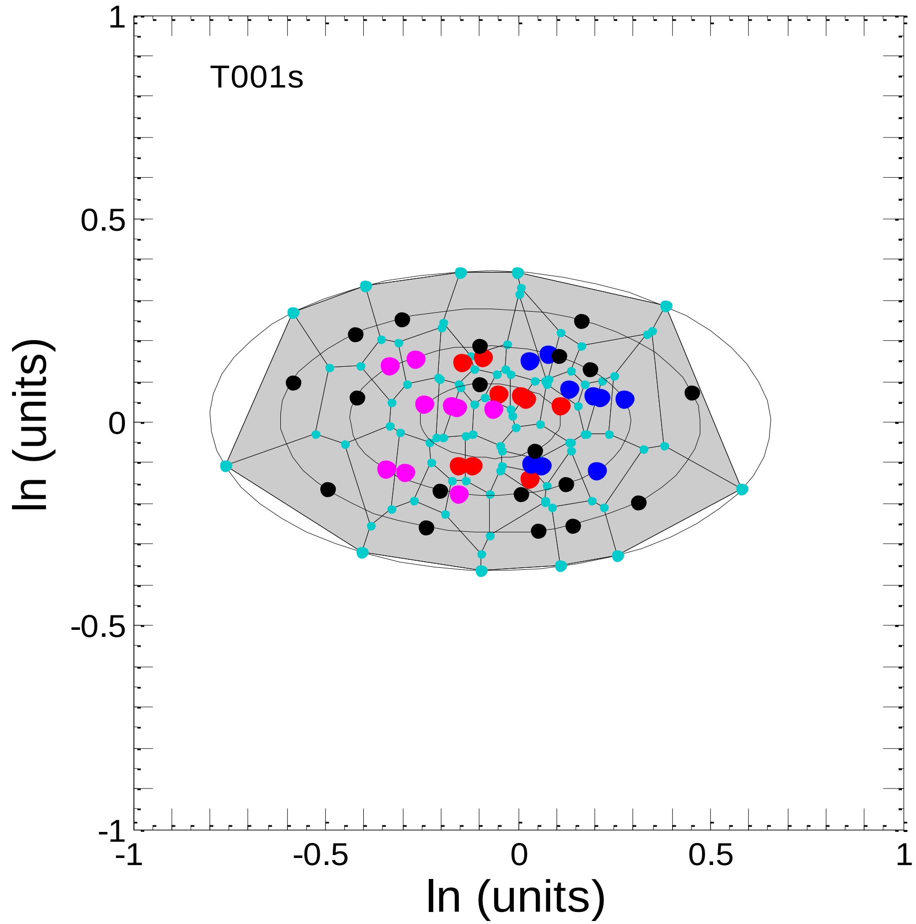
Selection of models – Approach 2





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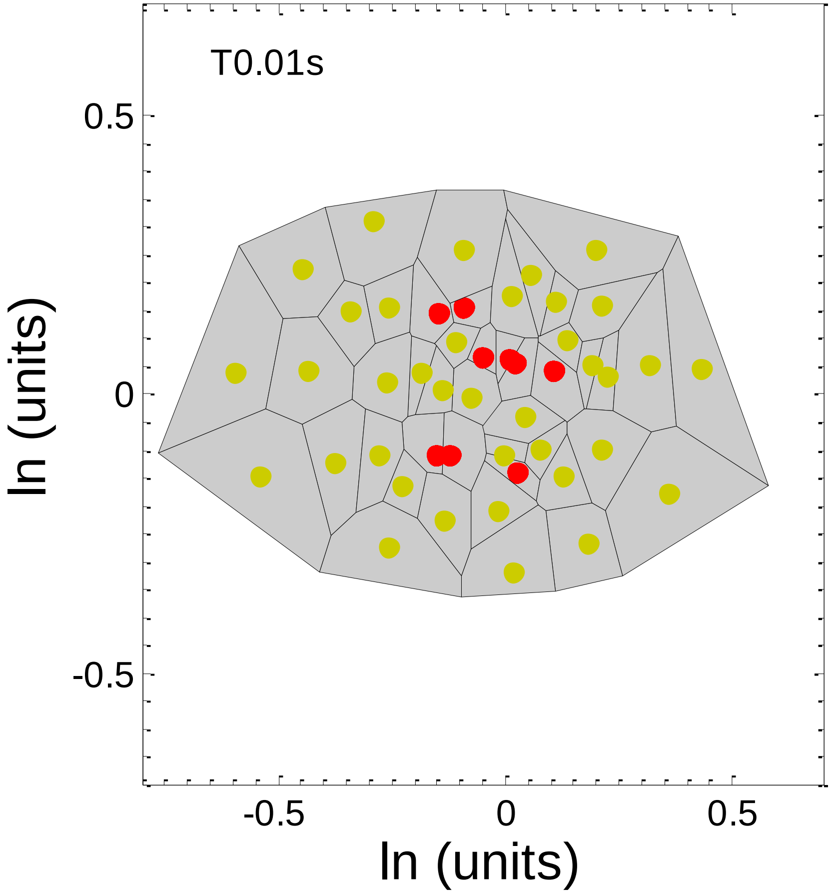
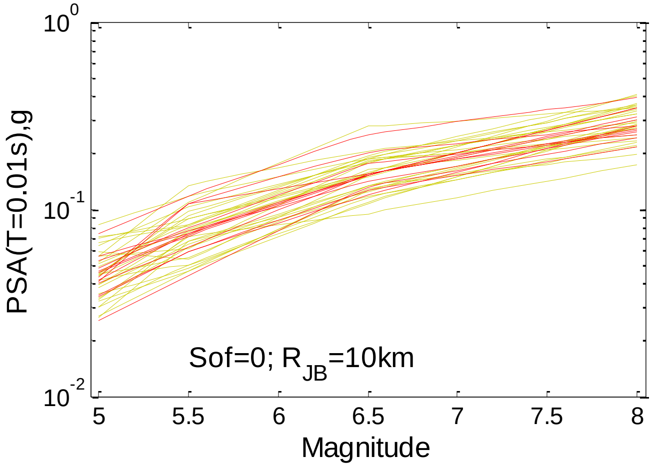
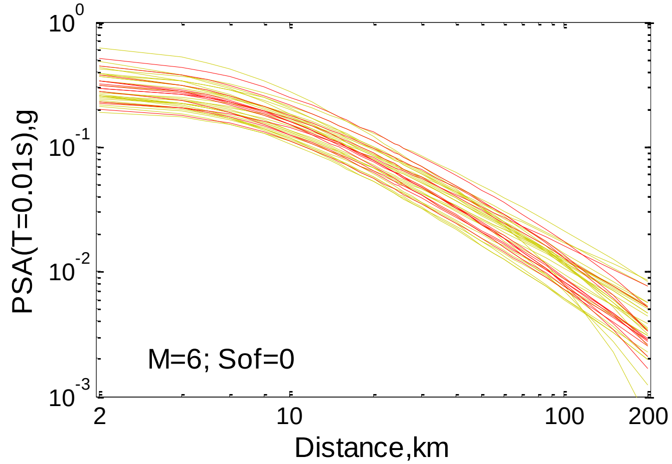
GMPE weightings and cells

- The Representative suite of common Form model
- The candidate GMPE models







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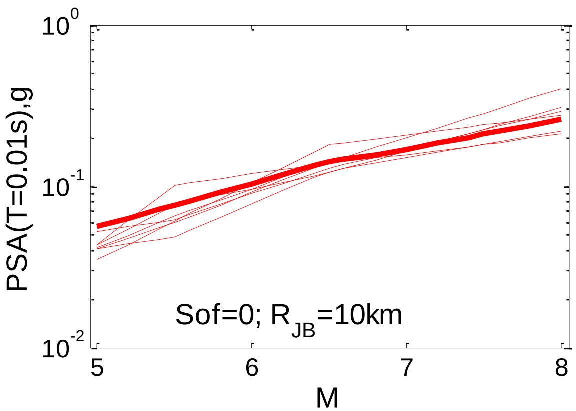
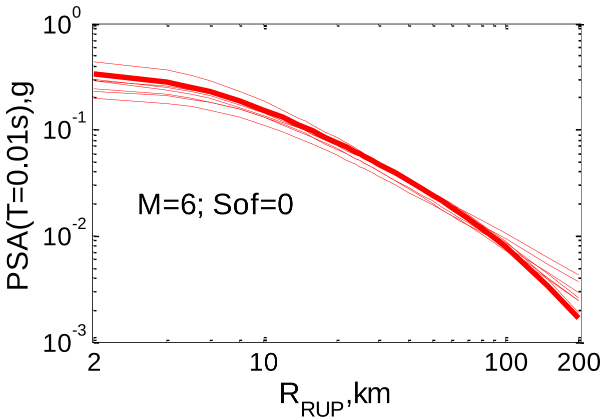
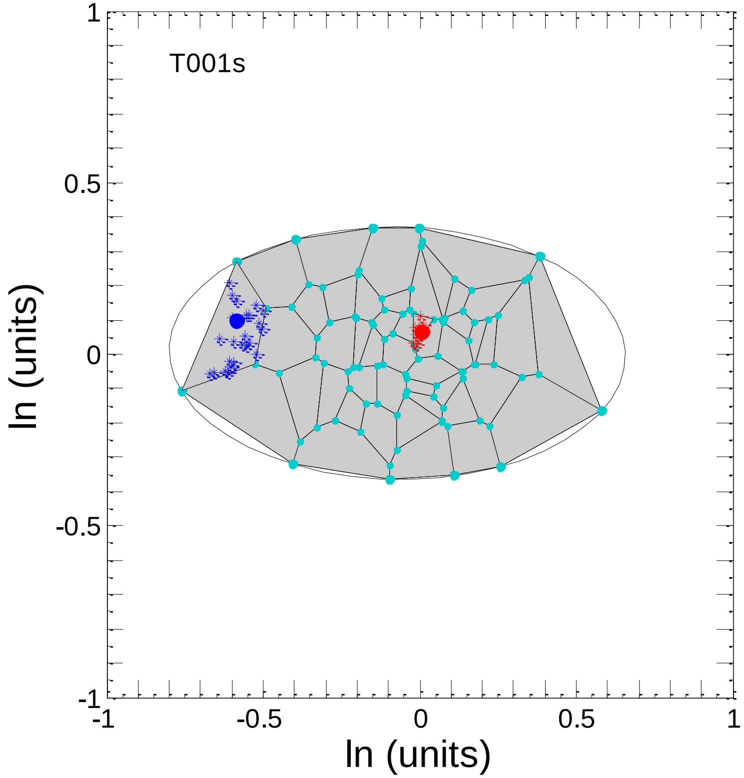
Selection of models – Approach 2





η	Log-likelihood
-0.0098	-280.873
-0.0895	-290.548
-0.1419	-310.989
0.0457	-284.204
0.0951	-299.791
0.0727	-292.808
0.0278	-276.865
0.0609	-289.394



These two values will change among different sub-region in the Voronoi Diagram

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GMPE weightings and cells

- **The weight for the selected representative model for each cell**

$$w_i = A_i \frac{1}{N_i} \sum_{j=1}^{N_i} L_{ji}$$

- $\boldsymbol{L_{ij}}$

  - $1/|\mu(\delta B)|$, One over the absolute mean between event residual.
  - $1/\mu(\delta B)^2$, One over the squared mean between event residual.
  - L, the likelihood.

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GMPE weightings and cells

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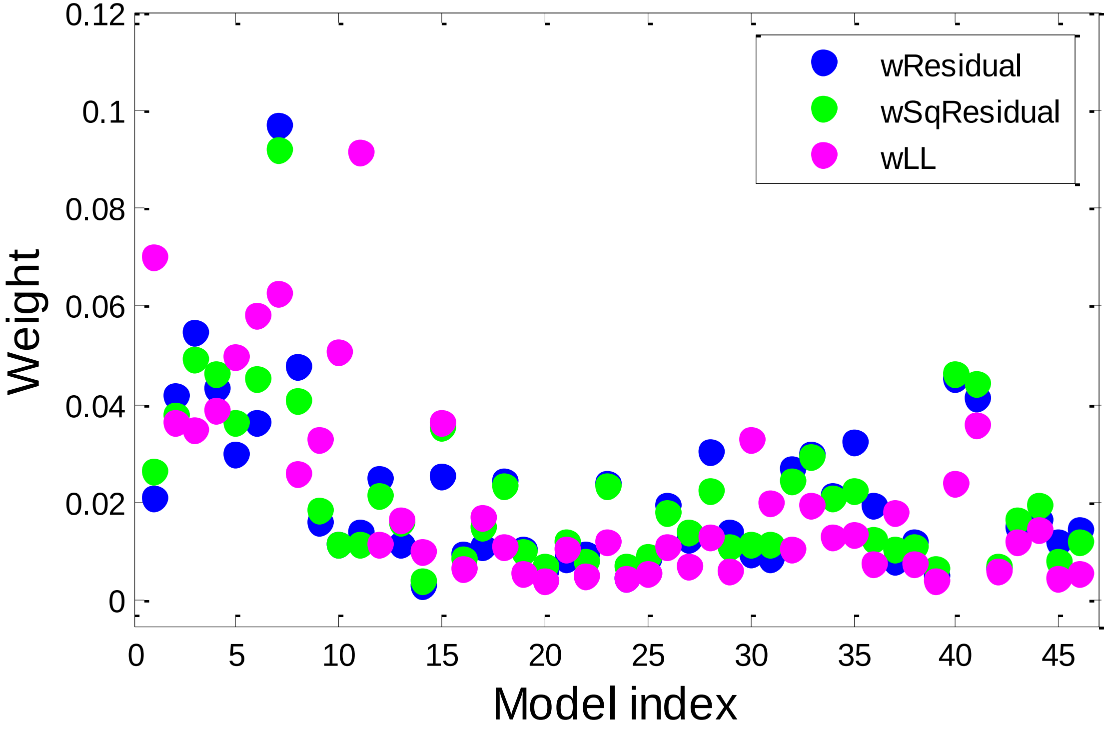

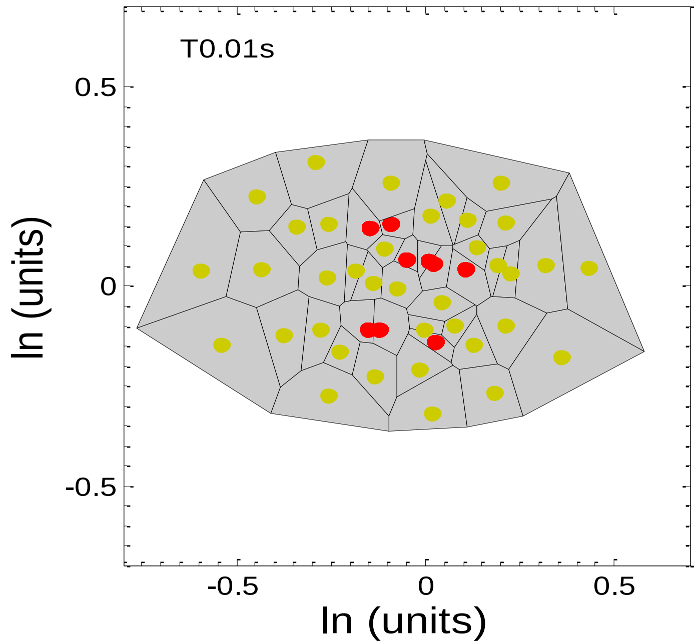

- *Capture **uncertainties in form & amplitude***
- *Conduct sensitivity analysis from the proposed weights on the seismic hazard*

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Conclusions

1. Method on the selection of GMPE for PSHA is introduced (The above–mentioned method had been used in "Diablo Canyon SSHAC Level-3 Report").

2. The calculated weighting value depends not only on the mean between event residual or likelihood value, but also depend on the area of each cell or the way of mapping is partitioned.

3. The proposed method can generate the quantitative value on selecting and ranking of GMPE model and provides information for experts on the judgment of weighting factor in seismic hazard calculation

Thank you for your attention !

Questions ?

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A Procedure of Logic Tree for GMPE

(to capture uncertainties in **form** as well as **amplitude**)

Selection of candidate GMPEs

- Identification of worldwide GMPEs
- Review of the GMPE applicability range
- Adjust for parameter compatibility
- Evaluation of the GMPE

Expert Judgment

- Logic tree from experts

Testing using data

- Ranking of GMPE

Proposition of logic trees

Sensitivity analysis of the proposed weights on the seismic hazard

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GMPE Distribution

Develop common funtional form for each candidate GMPE
$\mu(\ln y) = f(M, R, \theta, \ldots)$

For each GMPE *i*, estimate the set of coefficients θ_i (using senthetic data)

Calculate mean μ_θ and covariance Σ_θ

Given μ_θ and Σ_θ, sample new sets of coefficients $\{\theta\}$ and thus generate new models

In order to capture the correlation between the different coefficients θ, the common form is also fitted to the interpolated ground motions from the candidate GMPEs

$$Interp(\ln SA(M, R, Vs30)) = w_a \ln(SA_i) + w_b \ln(SA_j) \text{ with } i \neq j$$

$$\{w_a, w_b\} = \left\{\frac{1}{3}, \frac{2}{3}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{2}{3}, \frac{1}{3}\right\}$$

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Develop simple GMPE

- Simple functional form fit to Taiwan data:

$$\ln psa(\mathrm{T}\,0.01) = c_1 + c_{1a}FRV + c_{1b}\Delta Ztor + c_2M + c_3M^2 + c_4\log\left(R + c_5\exp\left(c_6M\right)\right)$$

$$\Delta Ztor = Ztor_i - \bar{Z}tor_{CY14}$$

  - Ztor is limited to 20km because we adopt Ztor-M relation of CY14.

- Method: Maximum regression using mixed – effect model

A STABLE ALGORITHM FOR REGRESSION ANALYSES USING THE RANDOM EFFECTS MODEL

Bulletin of the Seismological Society of America, Vol. 82, No. 1, pp. 505-510, February 1992

BY N. A. ABRAHAMSON AND R. R. YOUNGS

- Model bias and variability
  - $\tau =$ 0.4061.
  - ϕ = 0.5838.

c1	-6.3535
c1a	0.0855
C1b	0.0455
C2	2.0335
C3	-0.03826
C4	-2.0609
C5	0.388
C6	0.6268

Use this number to construct the log-likelihood contour

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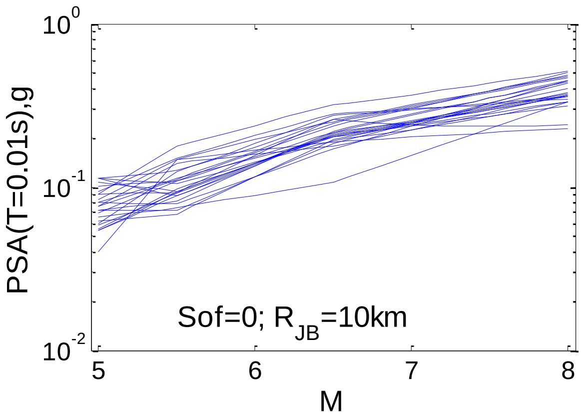
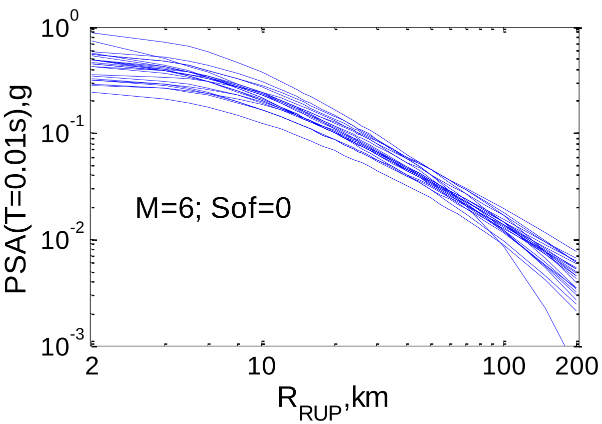
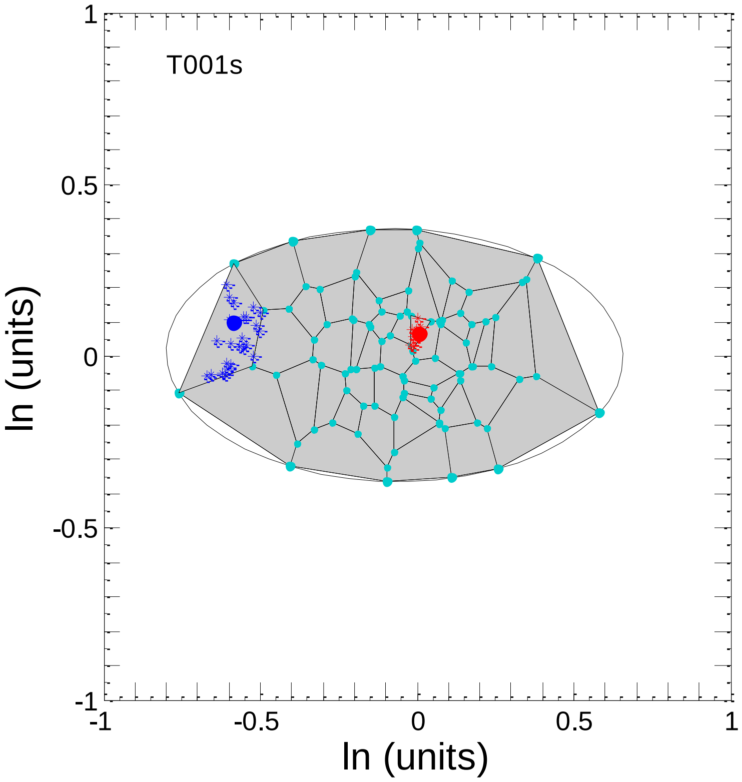
Selection of models – Approach 2





η	Log-likelihood
-1.023	-440.448
-1.0788	-399.806
-0.3595	-363.047
-0.7301	-419.606
-0.5533	-488.515
-0.6589	-474.253
-0.4225	-389.43
-0.5426	-390.071
-0.4222	-376.565
-0.2608	-323.921
-0.1877	-374.395
-0.1211	-304.776
-0.1739	-326.772
-0.3189	-335.055
-0.1726	-324.308
-0.0631	-295.312
-0.1776	-391.671
-0.1535	-330.265
-0.1453	-310.945
-0.076	-299.143
-0.1569	-329.054
-0.1148	-316.892
-0.2214	-325.172
-0.2319	-347.509



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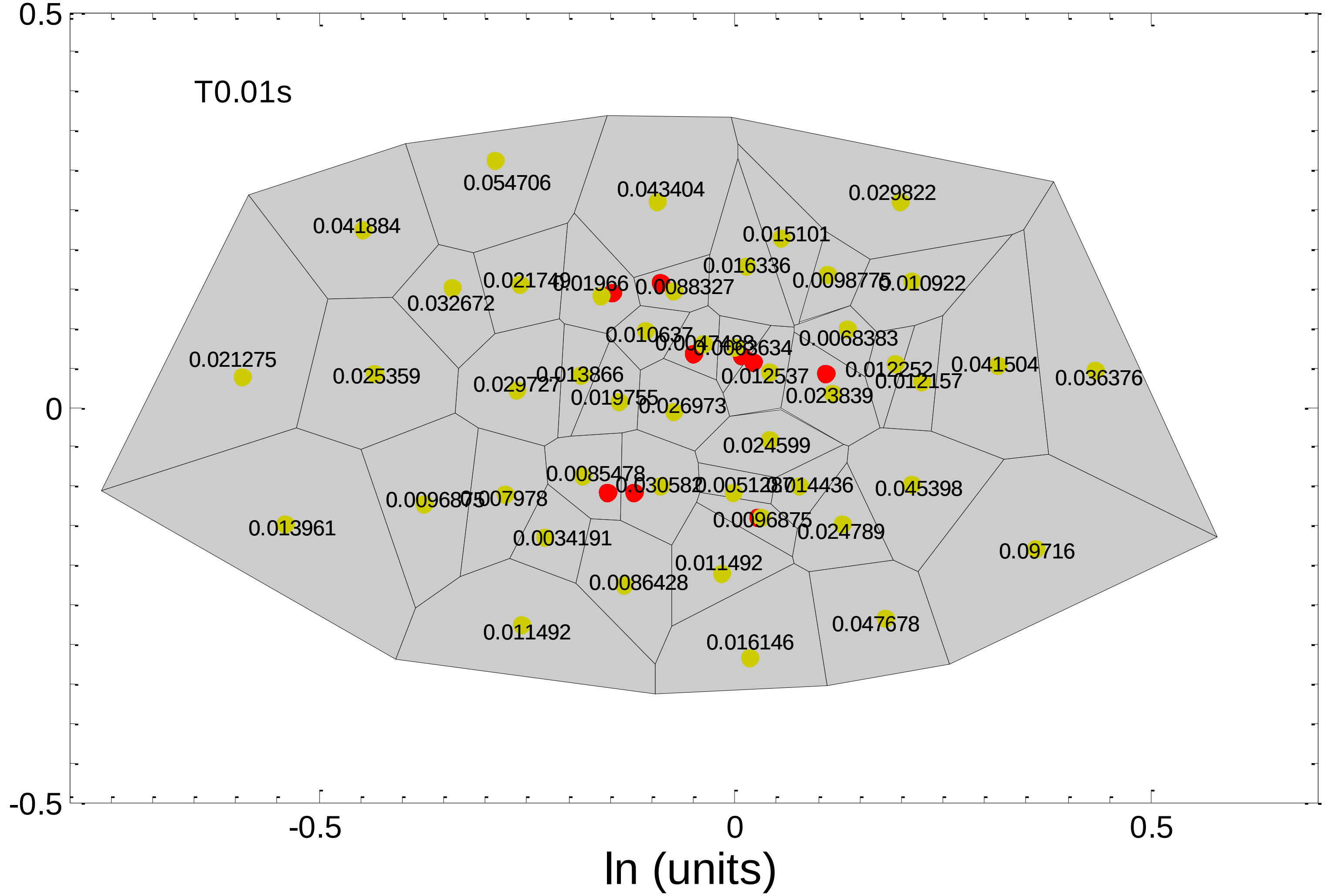
Distribution of GMPE weighting

- $1/|\mu(\delta B)|$, One over the absolute mean between event residual.



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