MODELING AND INVERSION OF THE MICROTREMOR H/V SPECTRAL RATIO:

THE PHYSICAL BASIS BEHIND THE DIFFUSE FIELD APPROACH

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5th IASPEI / IAEE International Symposium:

Effects of Surface Geology on Seismic Motion

Taipei, Taiwan, August 15-17, 2016

Outline

Microtremor H/V (MHVSR) → Site Characterization

Multiple Diffraction → Diffuse Fields → Coda ~ Noise

Correlations → Green's Function ← Equipartition & Isotropy

Auto-Correlations → Energy Densities ← Im $G_{11}(\mathbf{x}, \mathbf{x}, \omega)$

H/V from Ambient Seismic Noise is modeled as $\sqrt{2\mathrm{Im}G_{11}/\mathrm{Im}G_{33}}$

Elastic Im $G_{11}(0,0,\omega)$ behavior suggests simple 3D models

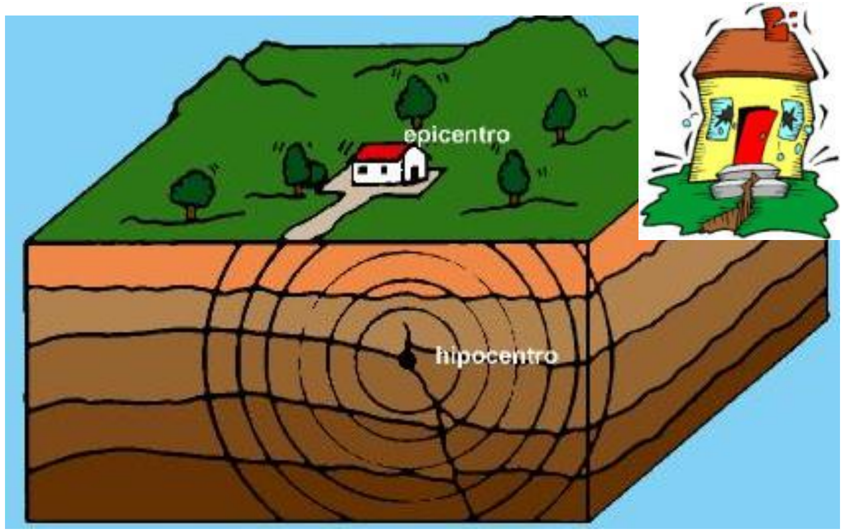
Inversion of H/V → Cauchy's theorem → Soil Information

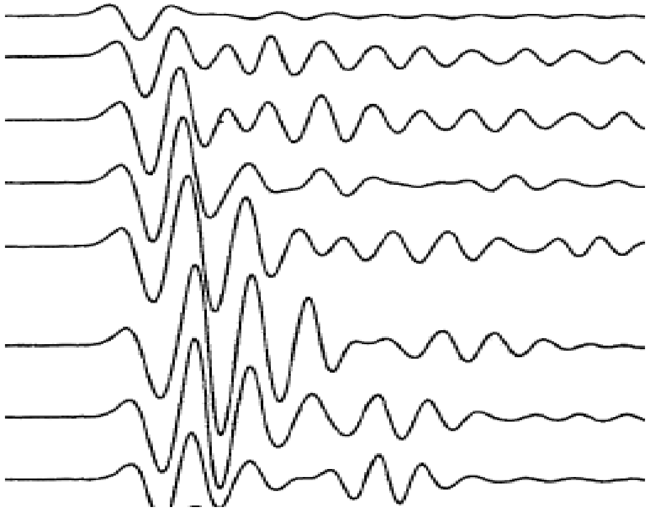
Joint inversion of H/V and DC → mitigates non-unicity

New software allows modeling and inversion of MHVSR.

Microtemor H/V Spectral Ratio MHVSR

Site Characterization → f_0 → Site effects





Amplification and Duration

Microtremor H/V Spectral Ratio MHVSR

- **Nogoshi & Igarashi (1971):**

  Microtremors → Surface Waves

- **Others Authors**

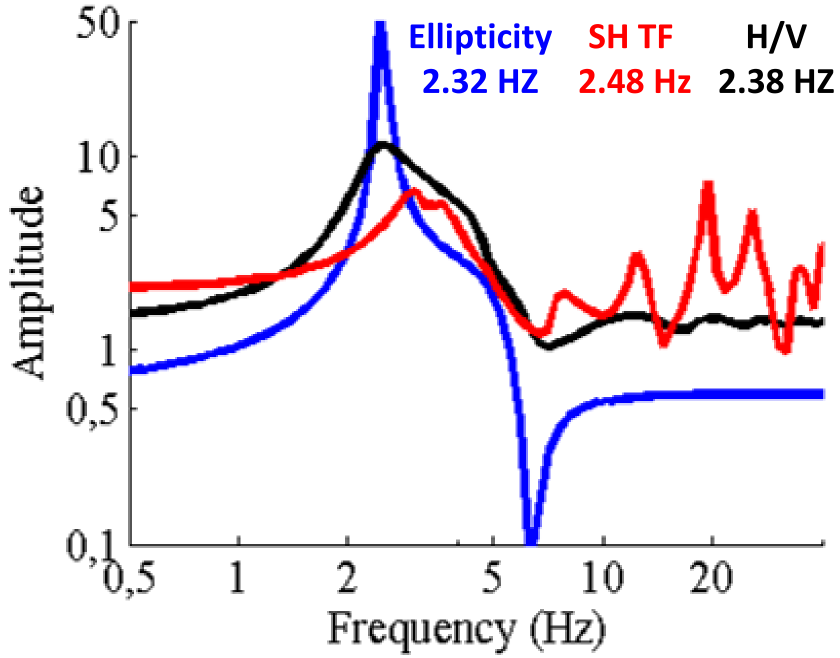
  ***H/V* ≡ Rayleigh Ellipticity**

- **Nakamura (1989)**

  ***H/V* ≡ Transfer Function SH wave**

$$\frac{H}{V} = \left\langle \left. \frac{H}{V} \right|_m \right\rangle$$

Average of Ratios



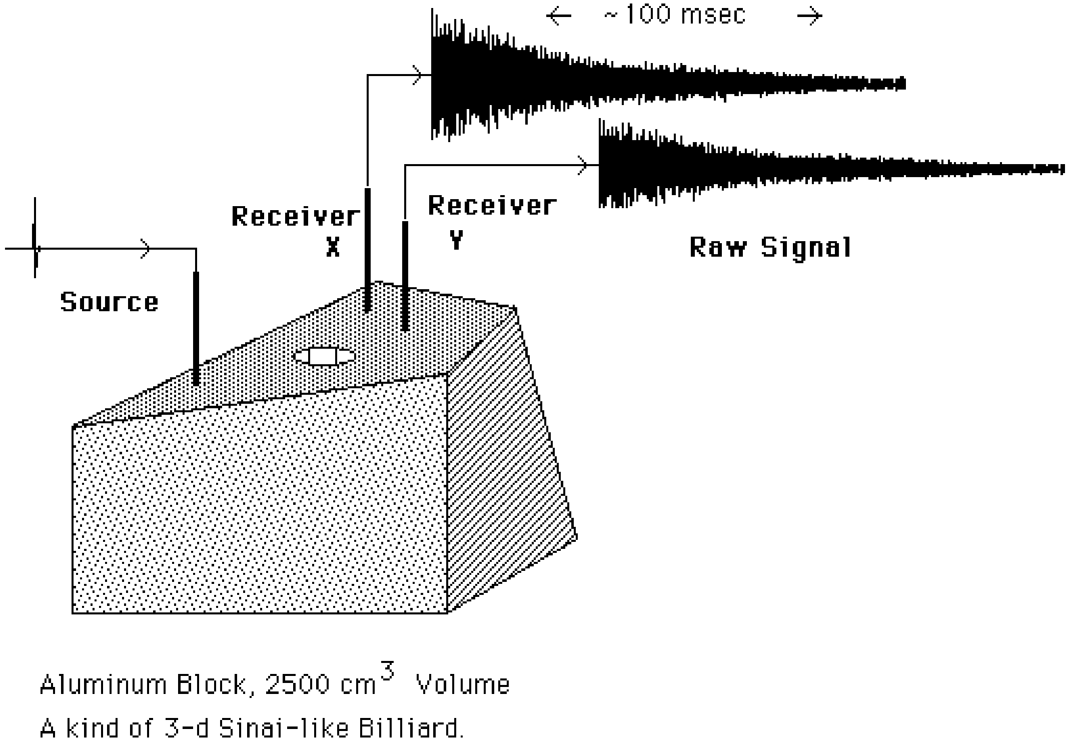
- **Arai & Tokimatsu (2004)**

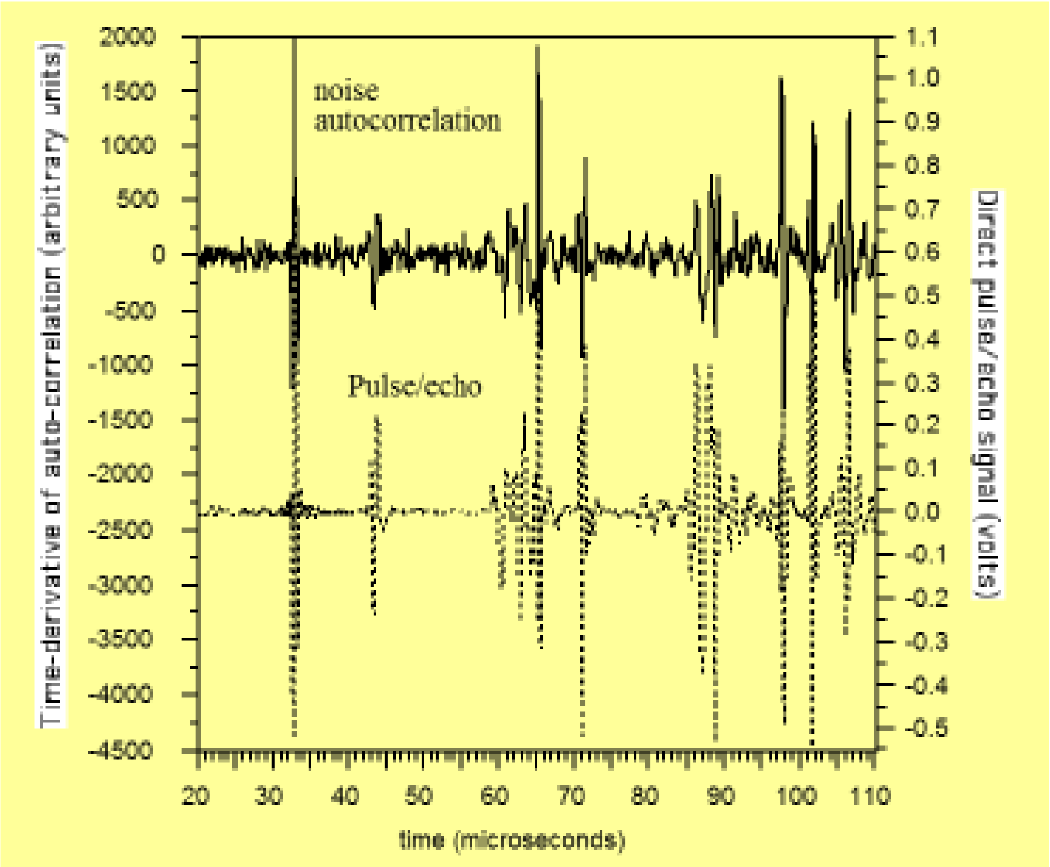
$$\frac{H}{V} = \sqrt{\frac{\langle H_{EWm}^2 \rangle + \langle H_{NSm}^2 \rangle}{\langle V_m^2 \rangle}}$$

Ratio of Averages

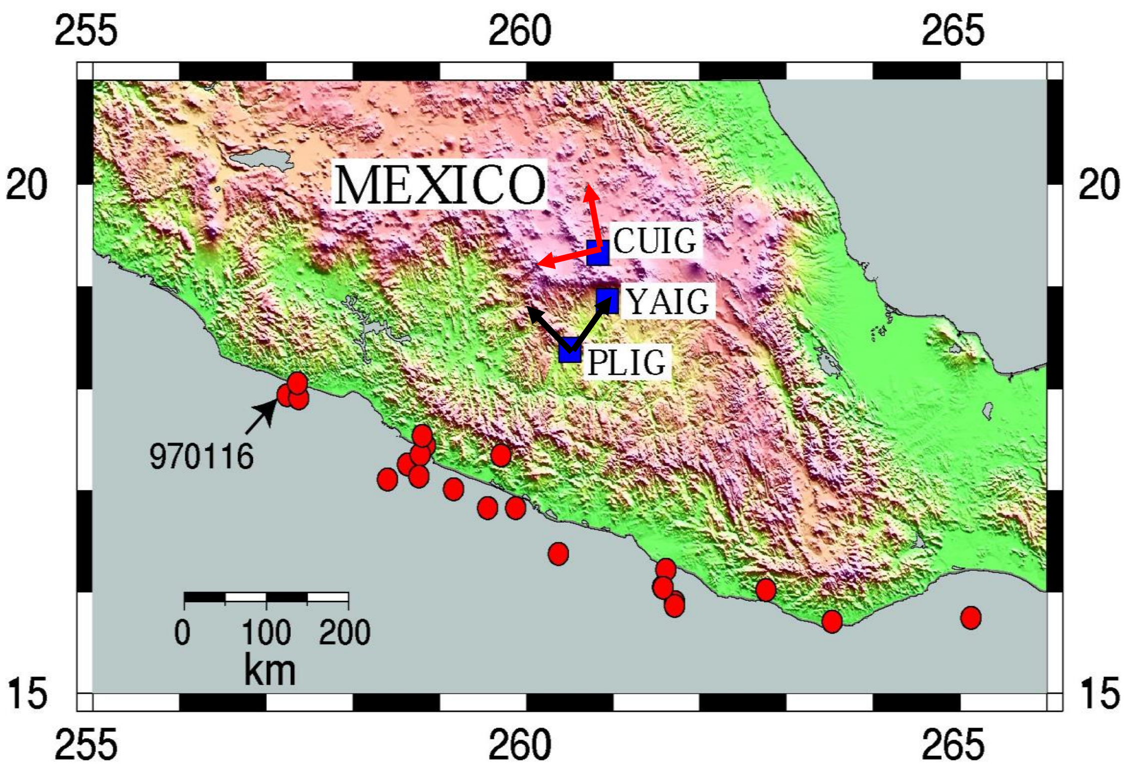
- **Sánchez-Sesma *et al.* (2011)**

  ***H/V* ≡ All Waves (Diffuse Field Theory)**

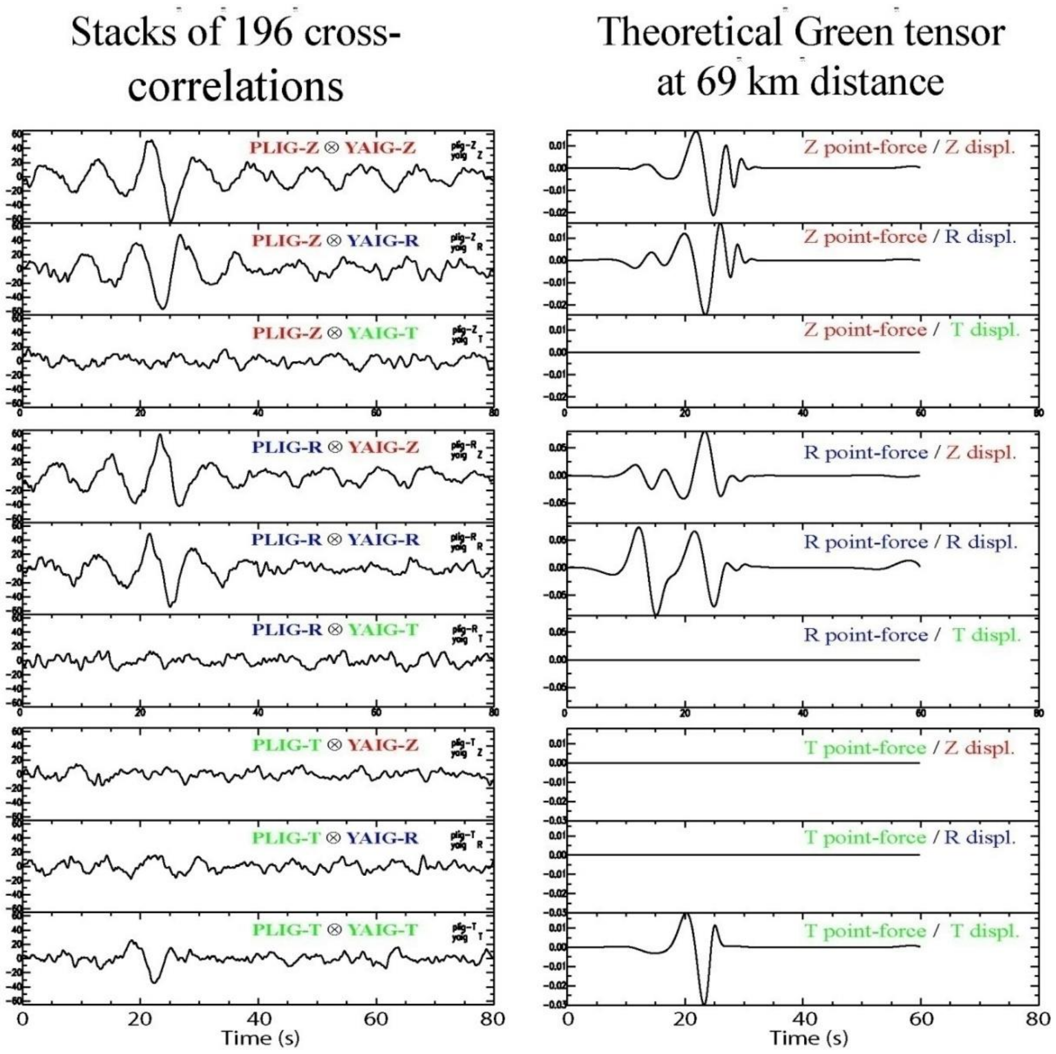




Weaver & Lobkis (2002) ***Ultrasonics***

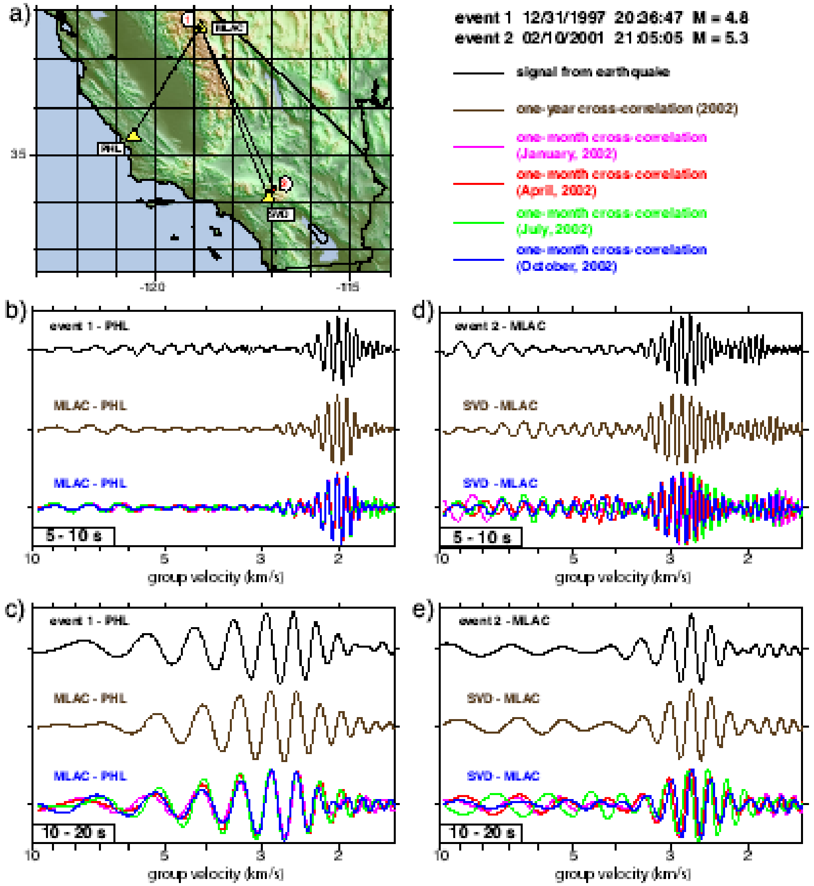


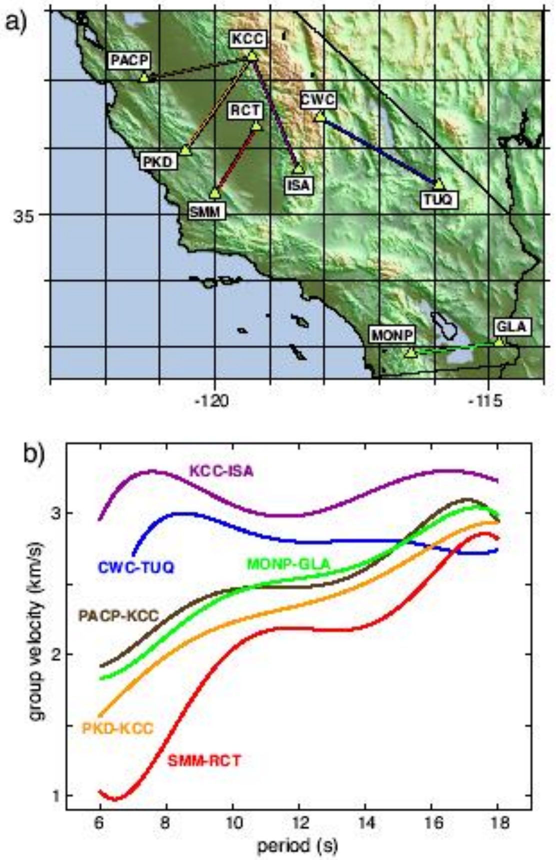
PLIG / YAIG : dist=69.6 km, az=41°
CUIG / YAIG: dist=53.1 km, az=167°

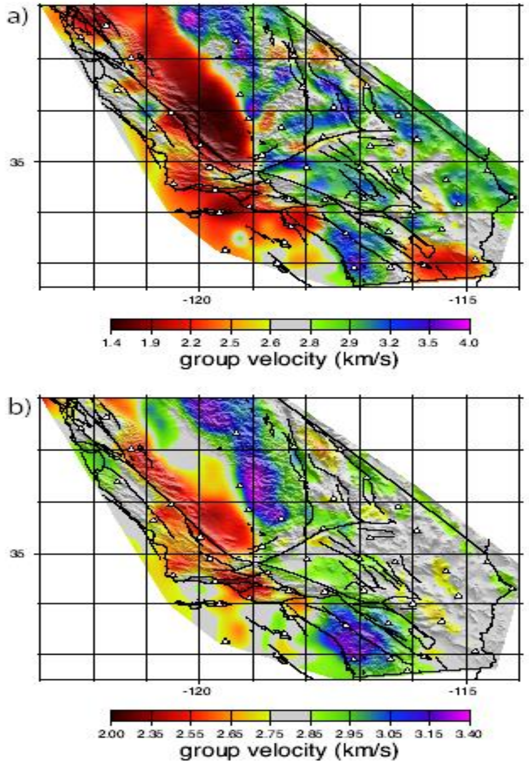


Campillo & Paul (2003) *Science*

Shapiro *et al* (2005) show waveforms emerging from cross-correlations of ambient seismic noise and compared them with Rayleigh waves excited by earthquakes



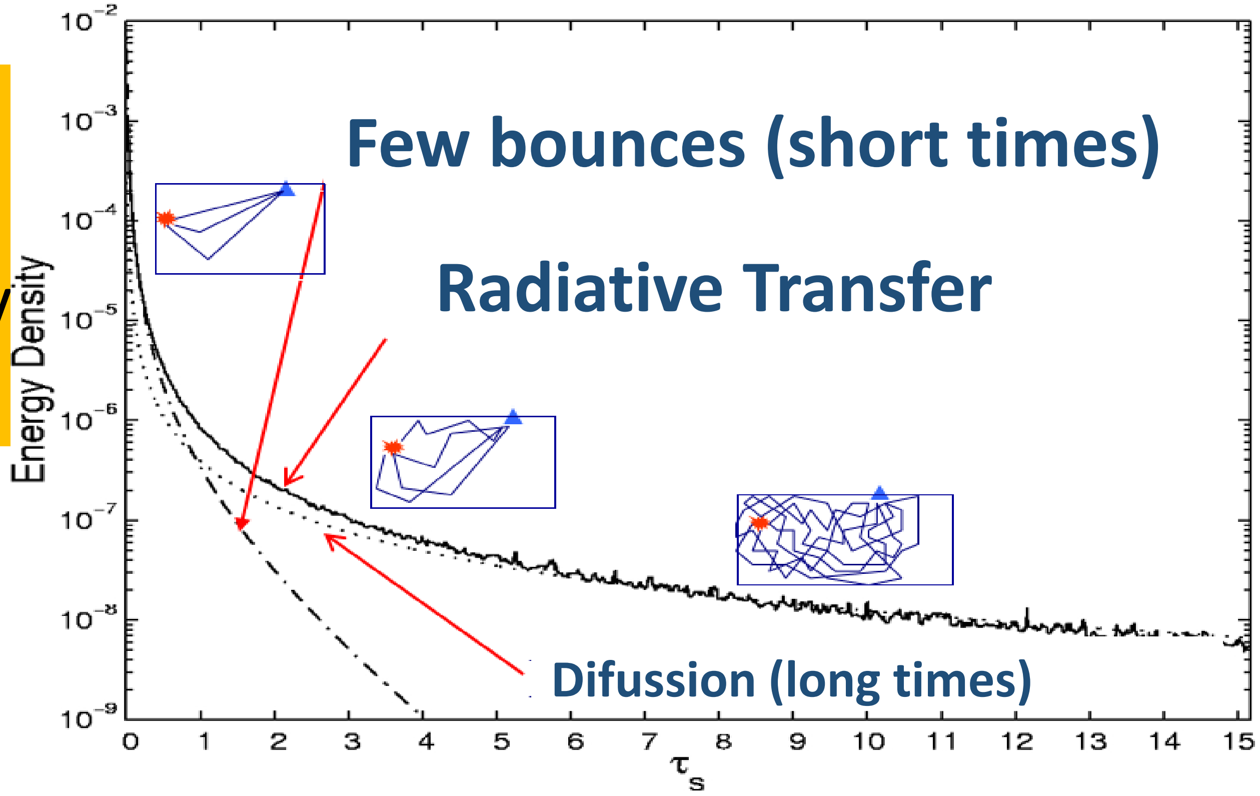




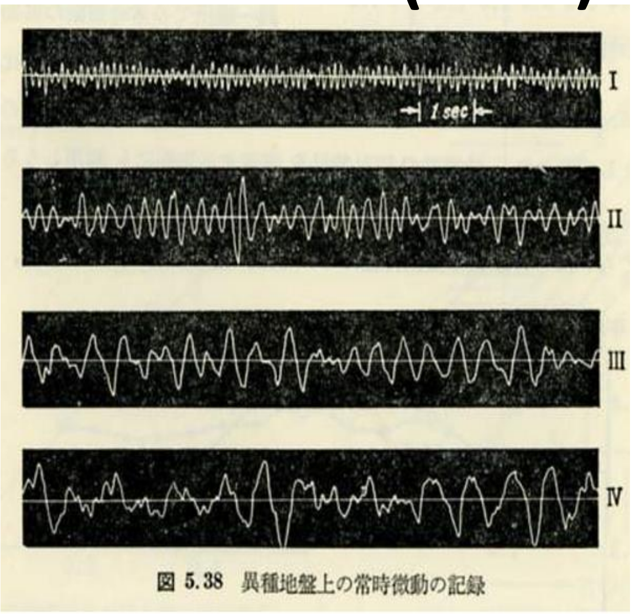
30 days of ambient noise. USArray

Multiple scattering Coda Noise Diffuse Field

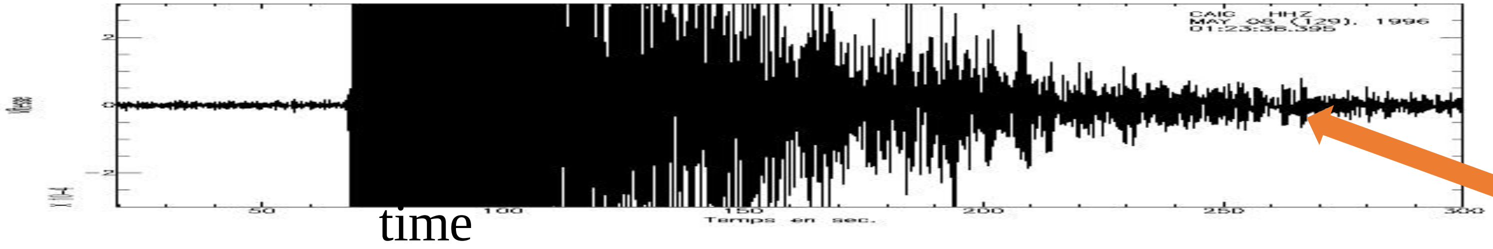
Propagation Regimes & Energy Density Decay



Kanai (1932)



Noise



Coda

Directional Energy Densities = Auto-Correlations → Im[$G(\mathbf{x},\mathbf{x},\omega)$]

$$\xi_1(\mathbf{x},\omega) = \rho\omega^2 \left\langle u_1(\mathbf{x})u_1^*(\mathbf{x}) \right\rangle = -2\pi\mu\xi_S k^{-1}\mathrm{Im}[G_{11}(\mathbf{x},\mathbf{x},\omega)]$$

Directional Energy Density (DED) is proportional to the imaginary part of Green's function at the source itself.

This is clear from the full-space solution (Stokes,1849):

$$\mathrm{Im}[G_{11}(\mathbf{x},\mathbf{x},\omega)] = \frac{-\omega}{12\pi\rho} \times \left\{ \frac{1}{\alpha^3} + \frac{2}{\beta^3} \right\}$$

$$\xi_1 = \xi_S \frac{\beta^3}{6\alpha^3}(1+2R^3) = \frac{\xi_P}{3} + \frac{\xi_S}{3} = \frac{\xi}{3}$$

$$\xi_P = \frac{1}{1+2R^3}\xi$$

$$\xi_{SH} = \frac{R^3}{1+2R^3}\xi$$

$$\xi_{SV} = \frac{R^3}{1+2R^3}\xi$$

$$\xi_1 = \frac{1}{3}\xi_P + \frac{1}{6}\xi_{SV} + \frac{1}{2}\xi_{SH} = \frac{1}{3}\xi$$

$$\xi_2 = \frac{1}{3}\xi_P + \frac{1}{6}\xi_{SV} + \frac{1}{2}\xi_{SH} = \frac{1}{3}\xi$$

$$\xi_3 = \frac{1}{3}\xi_P + \frac{2}{3}\xi_{SV} = \frac{1}{3}\xi$$

Weaver (1985) JASA

Sánchez-Sesma & Campillo (2006) BSSA

A Theory for H/V

With **Directional Energy Densities** the **H/V** ratio is:

$$[H/V](\mathbf{x};\omega)=\sqrt{\frac{E_1(\mathbf{x};\omega)+E_2(\mathbf{x};\omega)}{E_3(\mathbf{x};\omega)}}$$

$$[H/V](\mathbf{x};\omega)=\sqrt{\frac{\mathrm{Im}[G_{11}(\mathbf{x},\mathbf{x};\omega)]+\mathrm{Im}[G_{22}(\mathbf{x},\mathbf{x};\omega)]}{\mathrm{Im}[G_{33}(\mathbf{x},\mathbf{x};\omega)]}}$$

measurements $\longleftrightarrow$ system properties

Sánchez-Sesma et al. (2011)
3D problem (BW & SW)

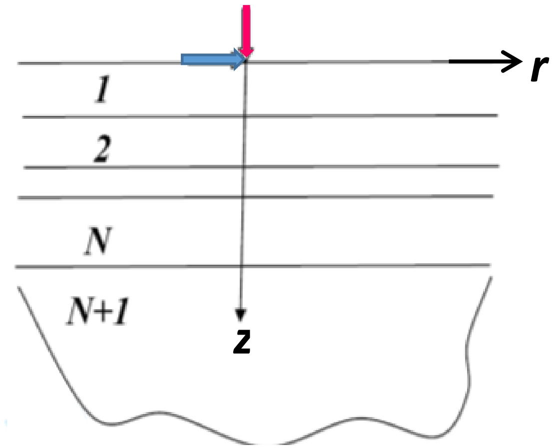
Kawase et al. (2011)
1D problem (BW)

Matsushima et al. (2014) 2.5D case (Lateral heterogeneity)

Lontsi et al. (2015) 1D H/V (*z*, ω) Data at depth

Green’s function calculation

The imaginary parts of the Green's functions, using **Harkrider (1964)** notation can be written as :

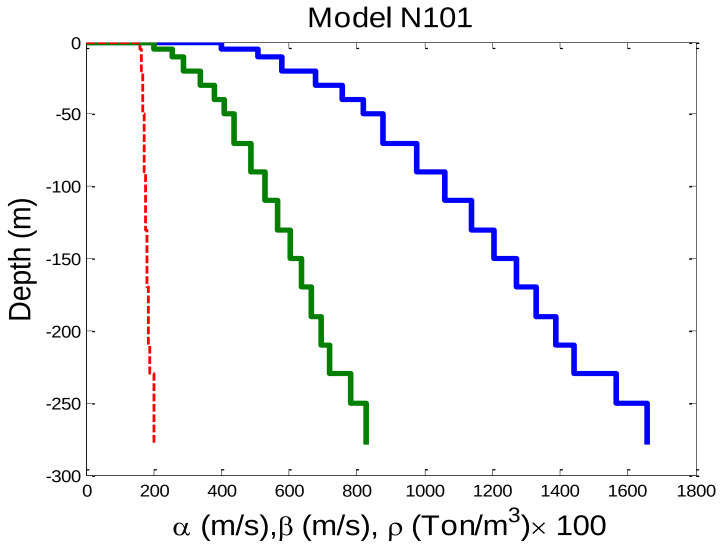


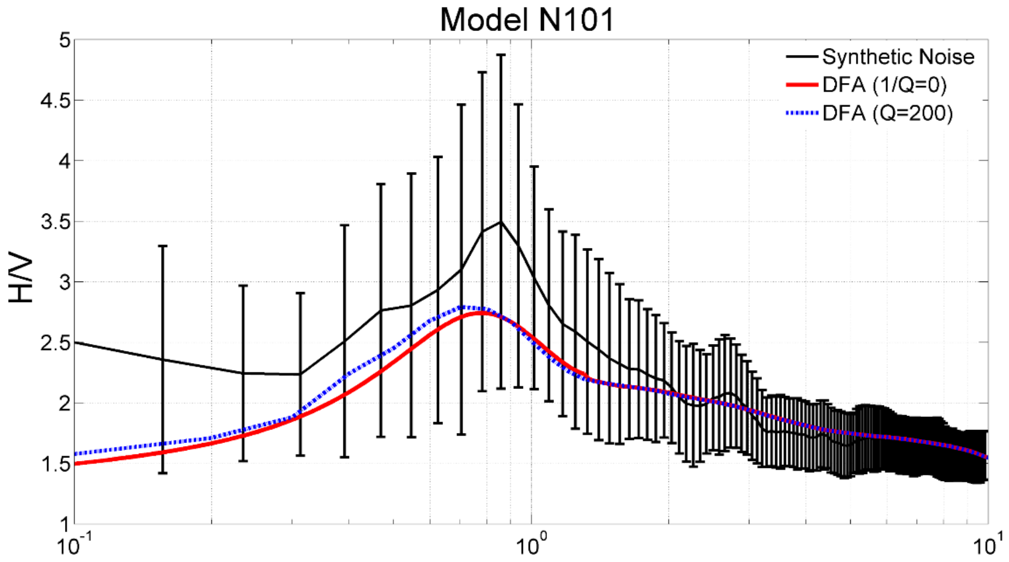
$$\mathrm{Im}[G_{33}(r;\omega)] = \mathrm{Im}\left[\frac{i}{2\pi}\int_0^\infty f_{P-SV}^{V}(k)J_0(kr)dk\right]$$

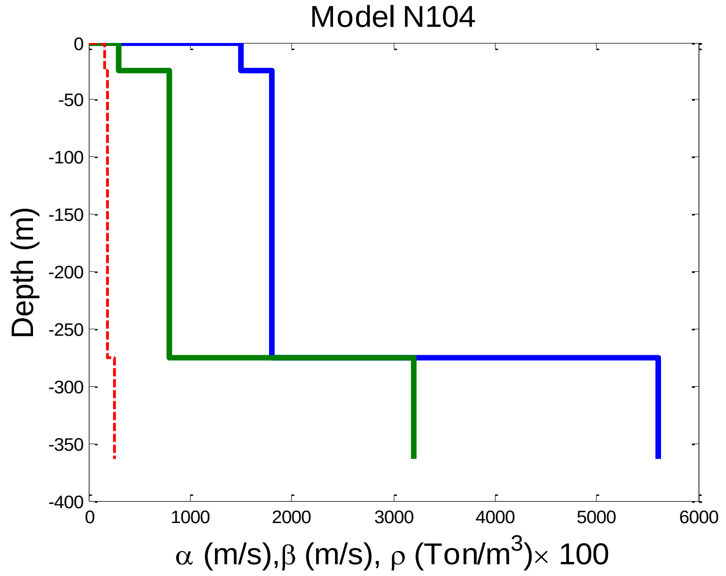
$$\mathrm{Im}[G_{22}(r;\omega)] = \mathrm{Im}[G_{11}(r;\omega)]$$

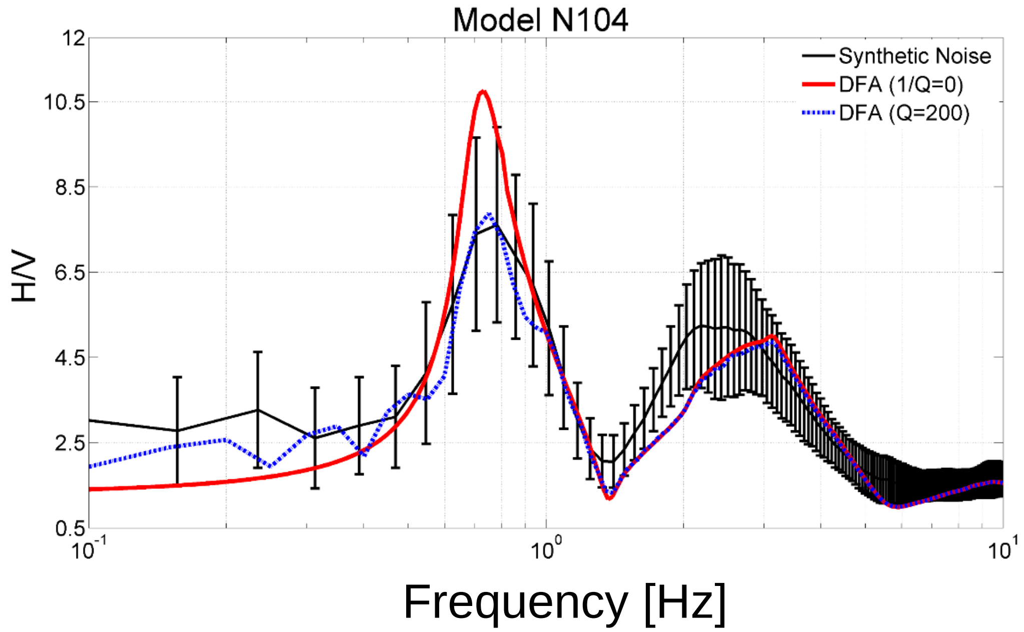
$$\mathrm{Im}[G_{11}(r;\omega)] = \mathrm{Im}\left[\underbrace{\frac{i}{4\pi}\int_0^\infty f_{SH}(k)[J_0(kr)+J_2(kr)]dk}_{SH} + \underbrace{\frac{i}{4\pi}\int_0^\infty f_{P-SV}^{H}(k)[J_0(kr)-J_2(kr)]dk}_{P-SV}\right]$$

Two blind tests (Synthetic Noise & DFA)

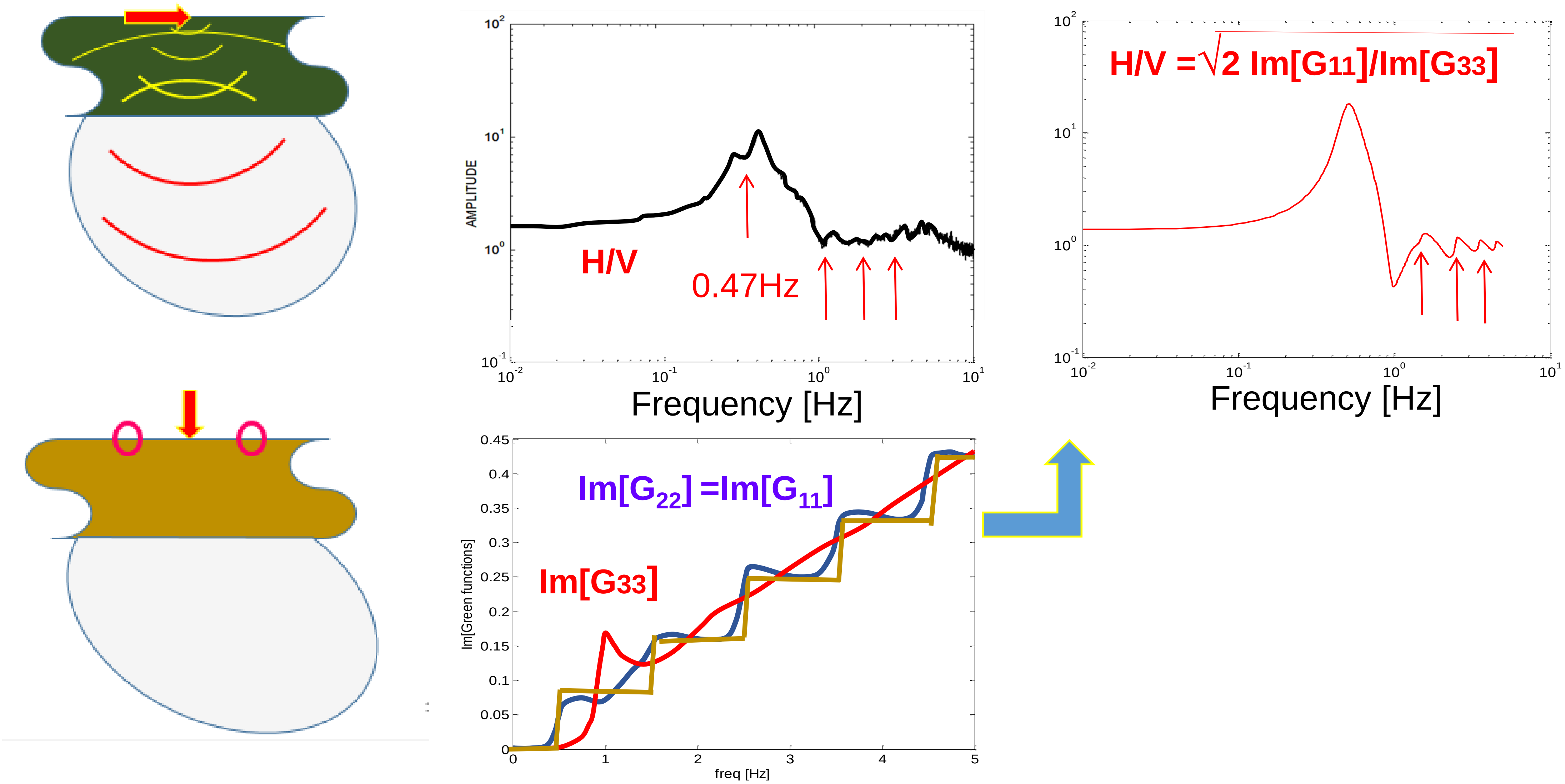




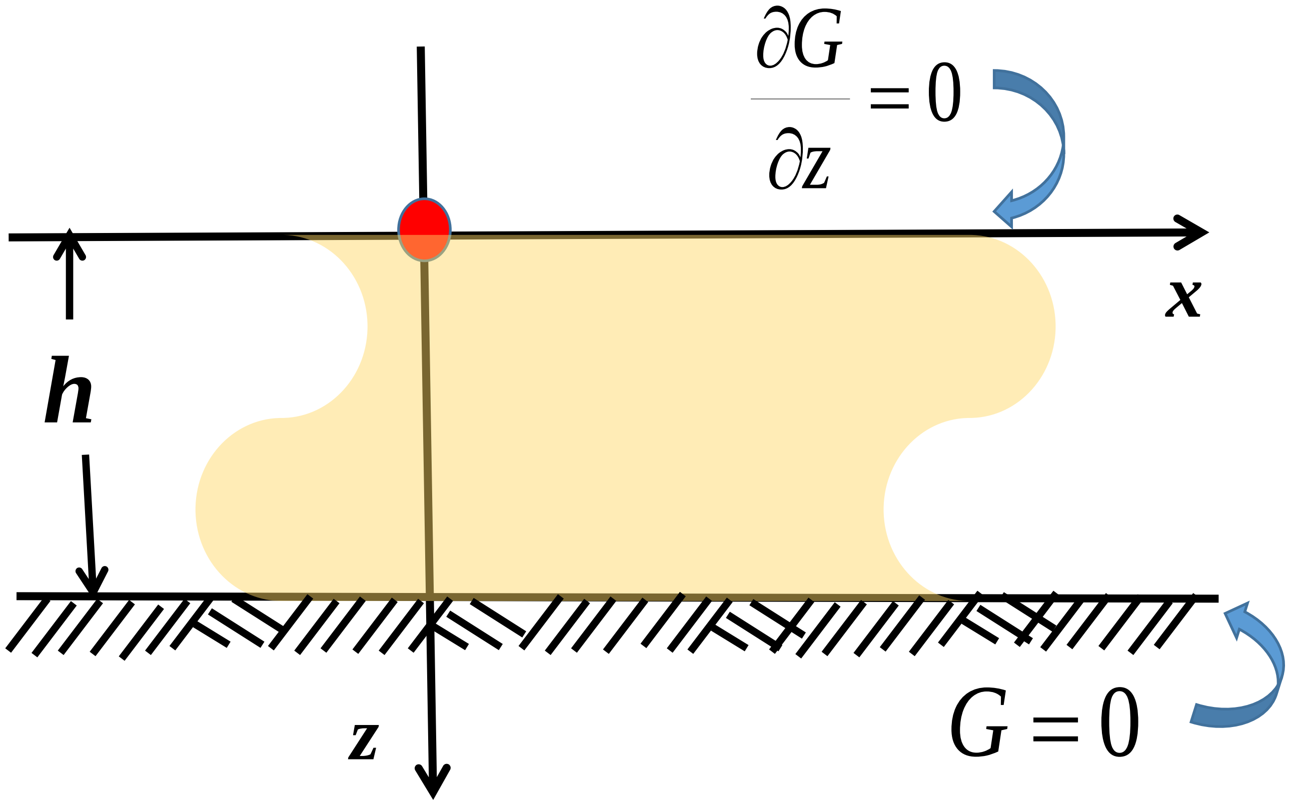




The Texcoco Experiment



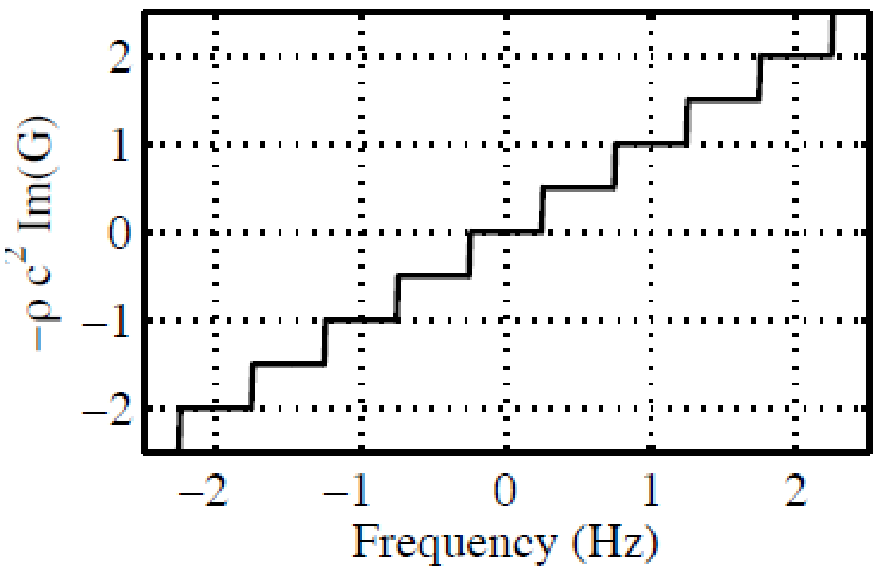
Simplified model in 3D



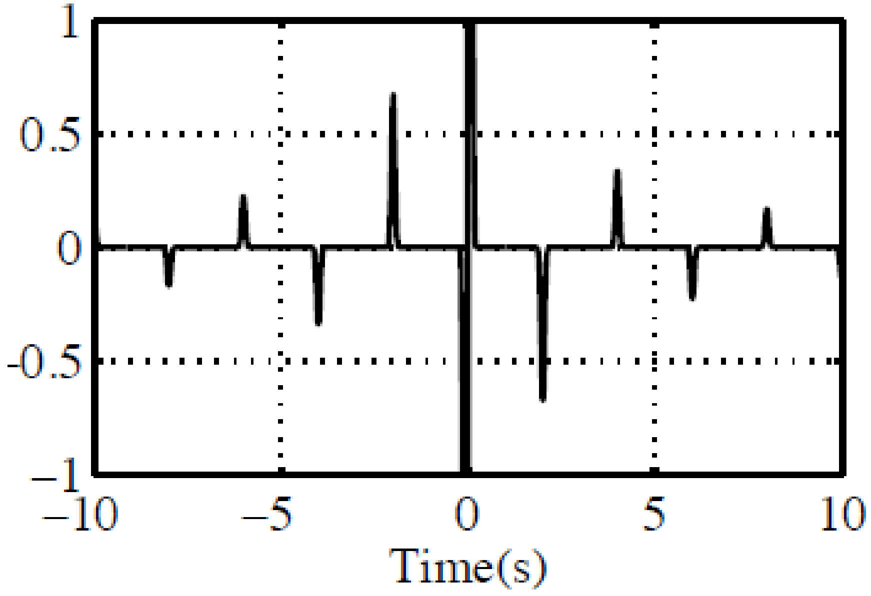
$$\nabla^2 G + k^2 G = -\frac{\delta(|\mathbf{x} - \xi|)}{\rho c^2}$$

Frequency and time domains 3D

$-\operatorname{Im} G(0,0,\omega)$



$PRS(0,0,t)$



$$\frac{1}{2\pi\rho c^2}\frac{\omega}{c}\sum_{n=0}^{\infty}(-1)^n\epsilon_n j_0(n\omega\tau),$$

$$\frac{1}{2\pi\rho c^2}\frac{\omega}{c}\sum_{n=0}^{\infty}(-1)^n\epsilon_n \mathrm{sinc}(n\omega\tau),$$

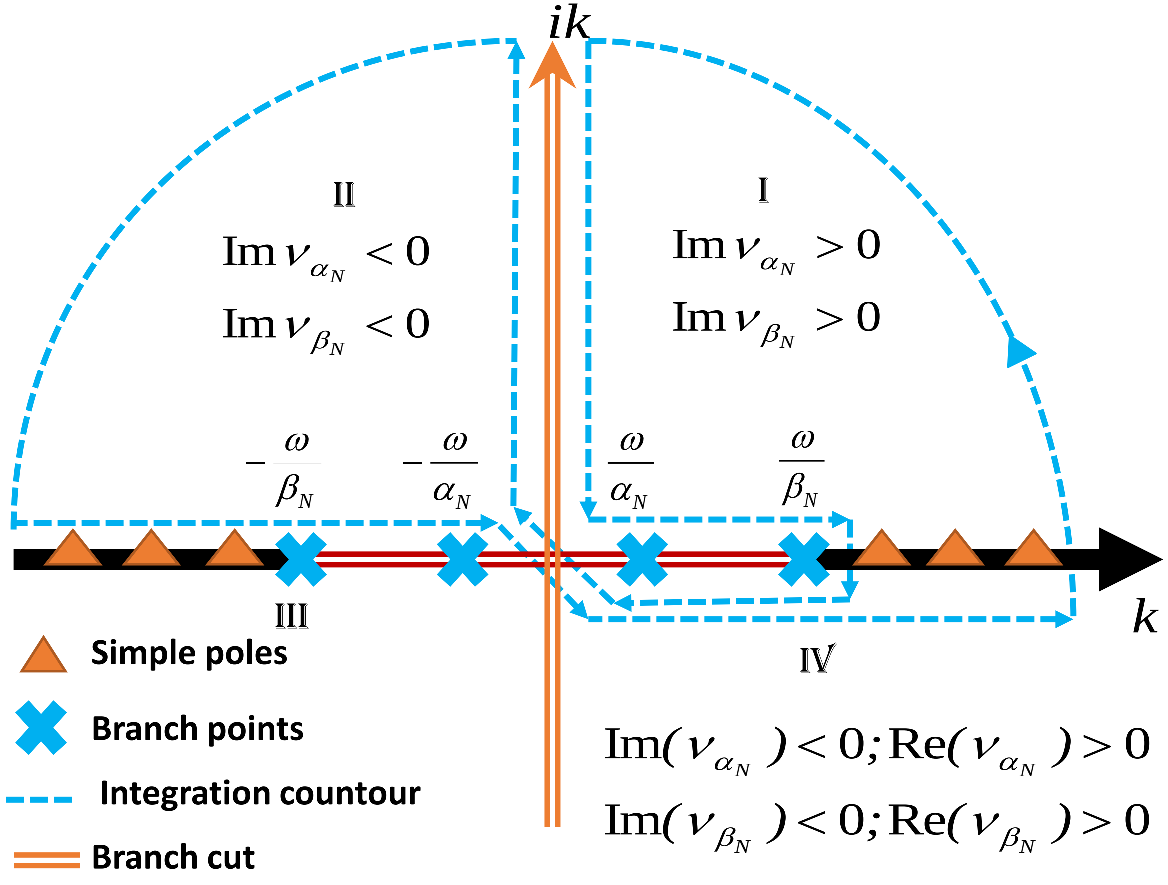
$$\frac{1}{2\rho c^2}\frac{1}{c}\sum_{n=0}^{\infty}H\left(\omega-\frac{(2n+1)\pi c}{2h}\right).$$

$$\frac{1}{2\pi\rho c^2}\frac{1}{2h}\left\{-\frac{d\delta(t)}{dt}\tau+\sum_{n=0}^{\infty}\frac{(-1)^n}{n}\delta(|t|-n\tau)\,\mathrm{sgn}(t)\right\}$$

Green's function calculation

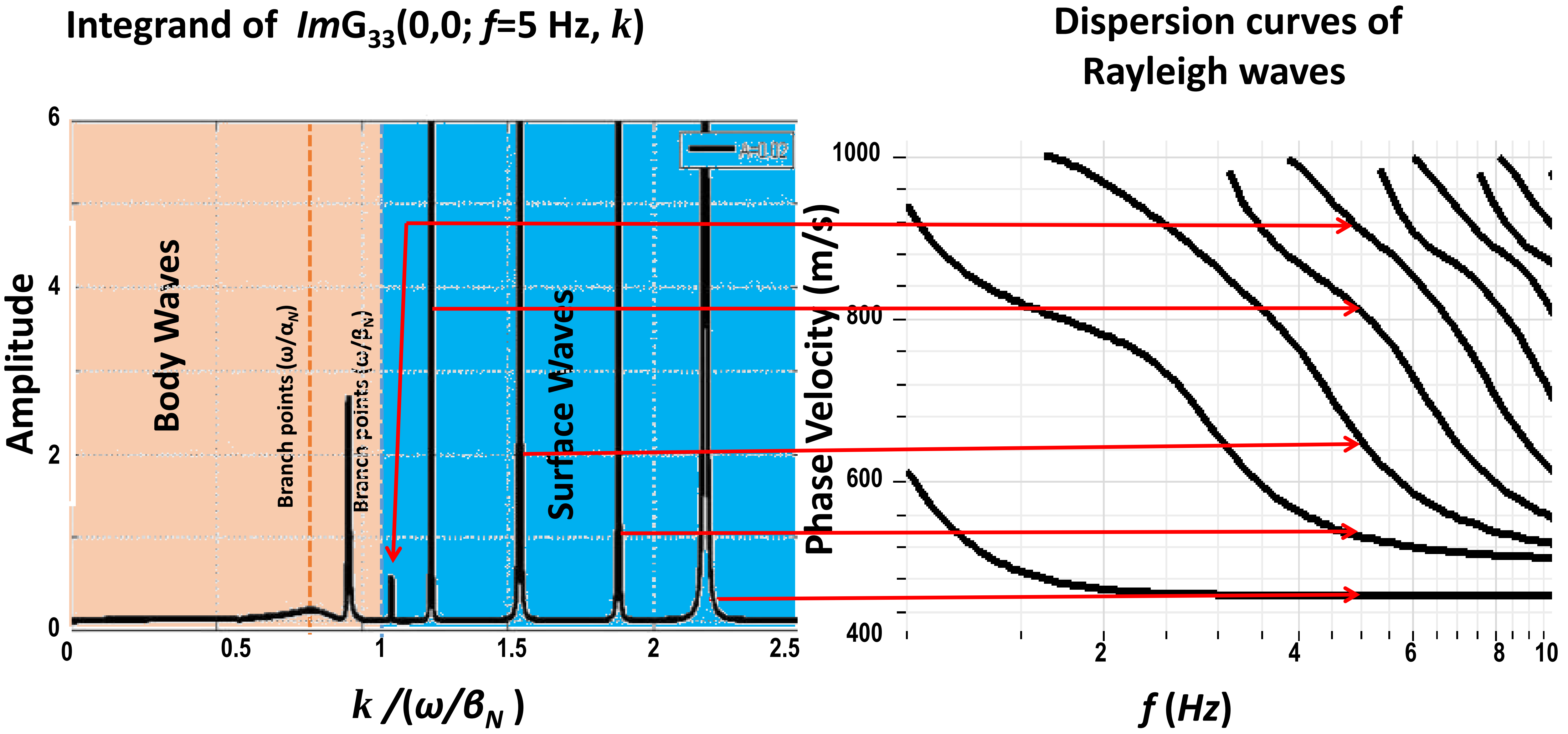
Cauchy's Residue theorem

The integrand of the Green's function has simple poles isolated on the real axis *k* and branch points ω/β y ω/α.



Augustin-Louis Cauchy

Position of poles



Green function calculation

The imaginary parts of the Green's function is computed as **(García-Jerez et al., 2013):**

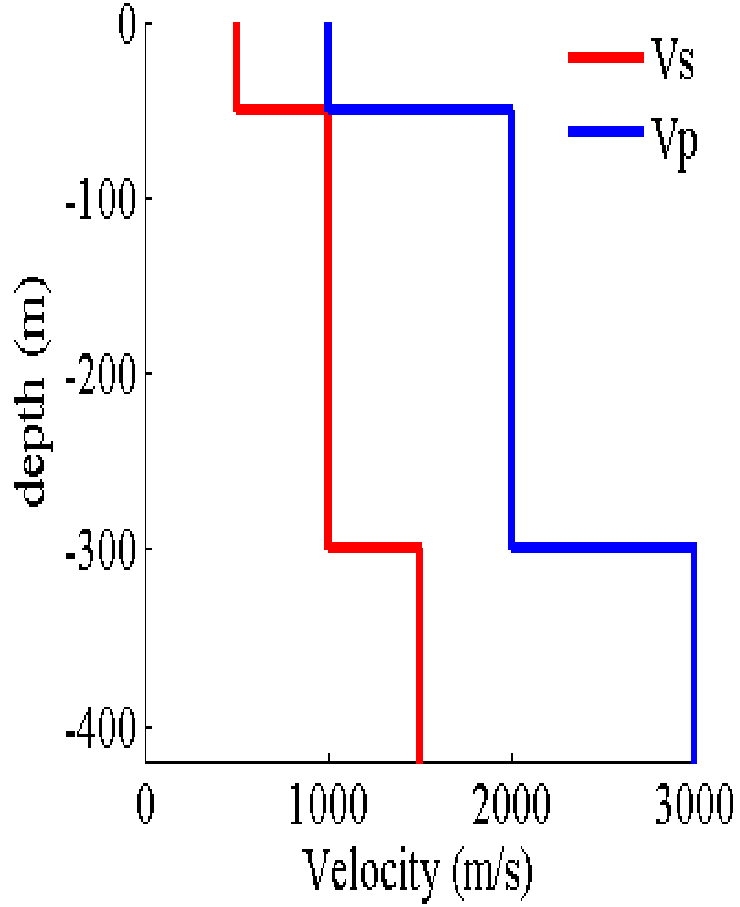
$$\operatorname{Im}\left[G_{11}(\omega)\right] = \underbrace{-\frac{1}{4}\left[\sum_{m\in\text{Rayleigh}} \chi_m^2 A_{Rm} + \sum_{m\in\text{Love}} A_{Lm}\right]}_{\text{Surface Waves}} + \underbrace{\frac{1}{4\pi}\int_0^{\omega/\beta_N} \operatorname{Re}\left(\left[f_{P-SV}^{H}(k)\right]_{4^{th}} + \left[f_{SH}(k)\right]_{4^{th}}\right) dk}_{\text{Body Waves}}$$

$$\operatorname{Im}\left[G_{22}(\omega)\right] = \operatorname{Im}\left[G_{11}(\omega)\right]$$

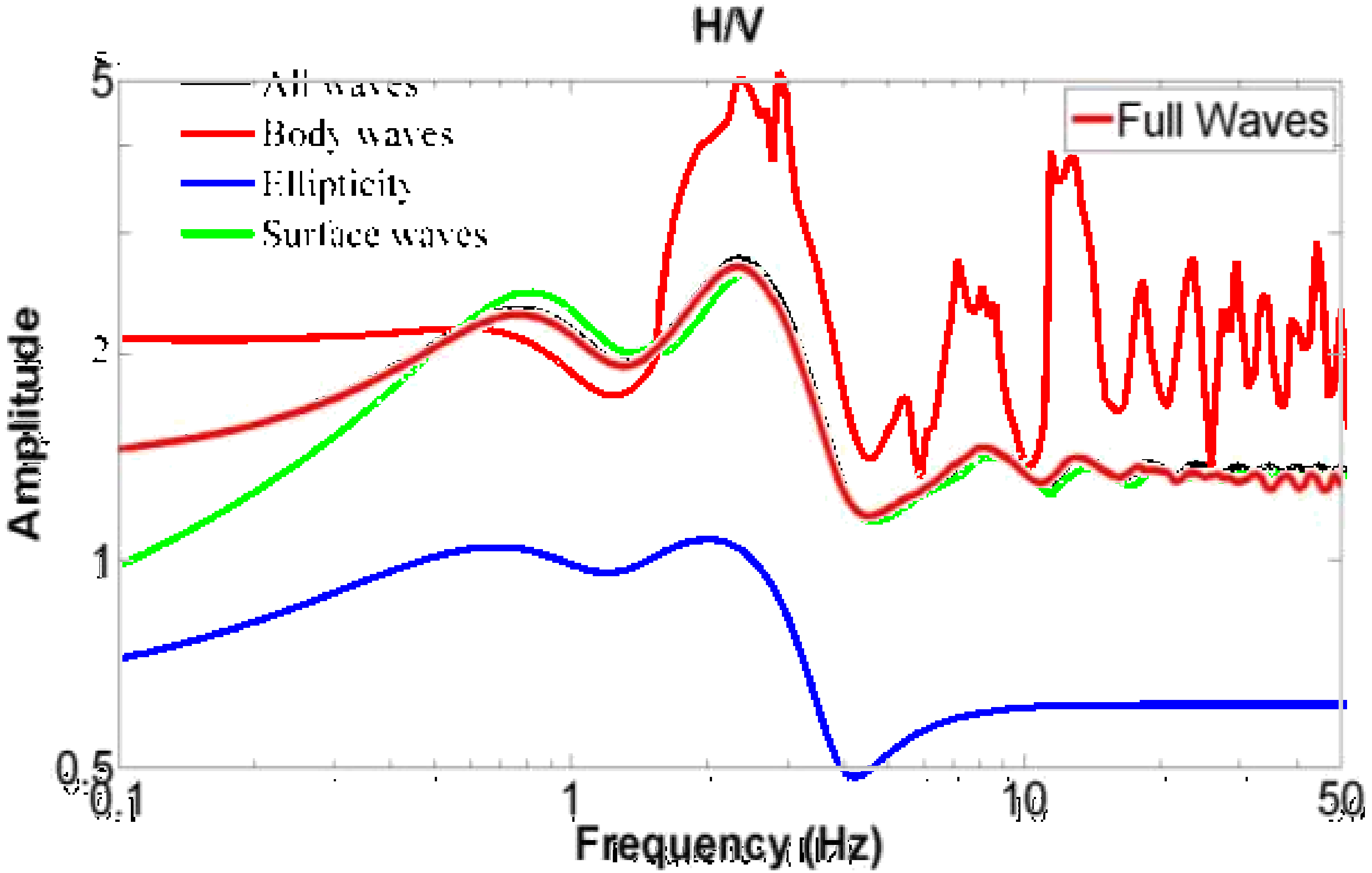
$$\operatorname{Im}\left[G_{33}(\omega)\right] = \underbrace{-\frac{1}{2}\sum_{m\in\text{Rayleigh}} A_{Rm}}_{\text{Surface Waves}} + \underbrace{\frac{1}{2\pi}\int_0^{\omega/\beta_N} \operatorname{Re}\left[f_{P-SV}^{V}(k)\right]_{4^{th}} dk}_{\text{Body Waves}}$$

Several views of H/V

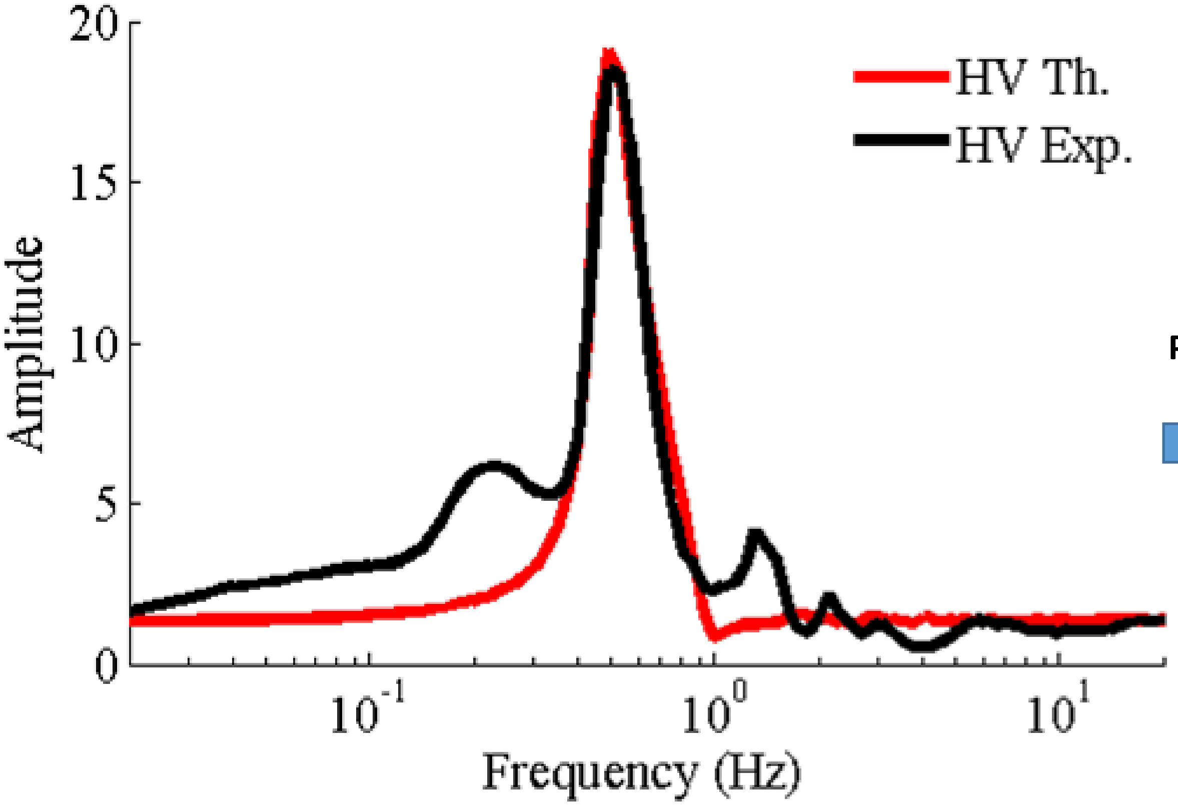
Profile Velocity



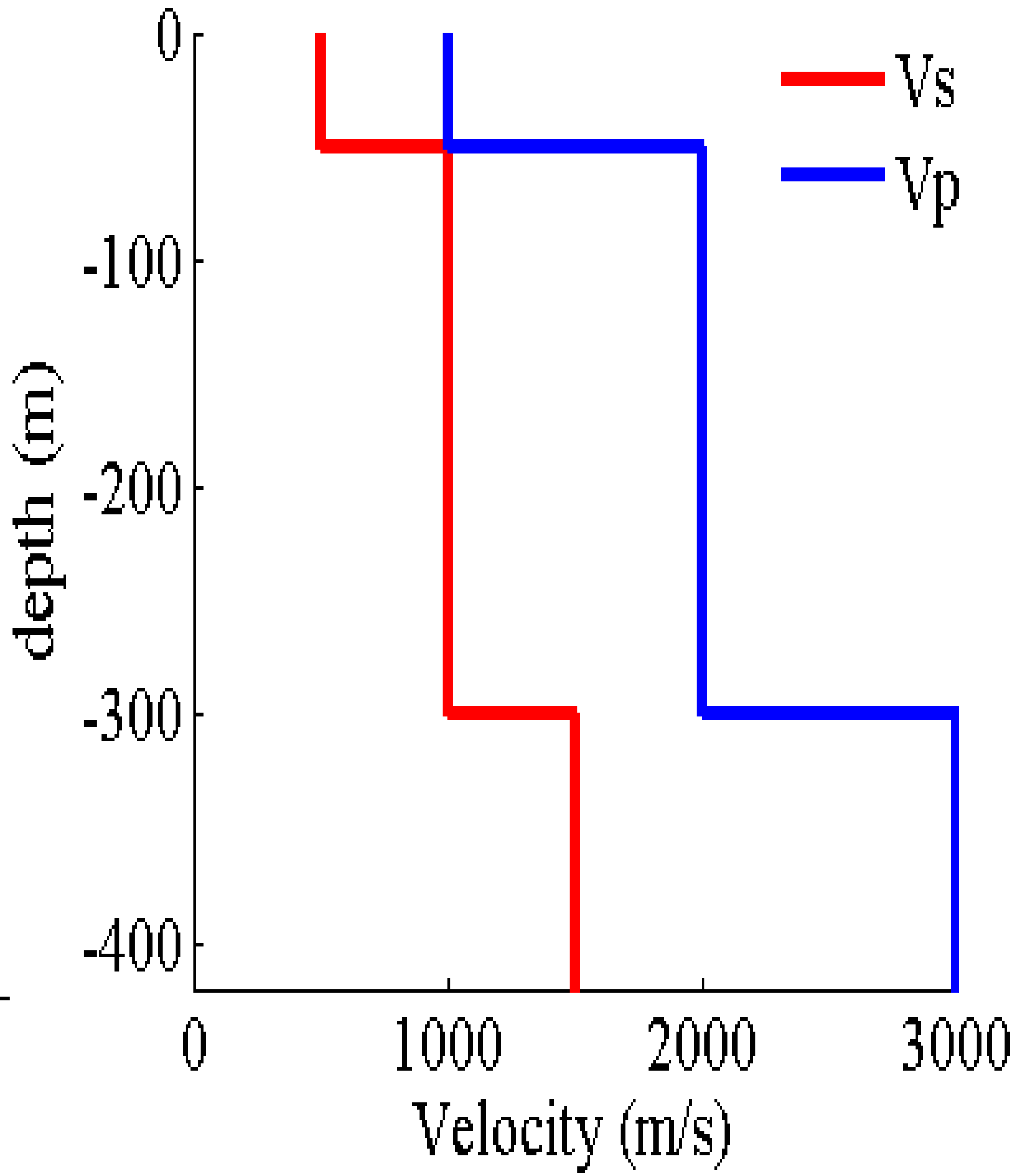
Foward Problem



Inverse Problem

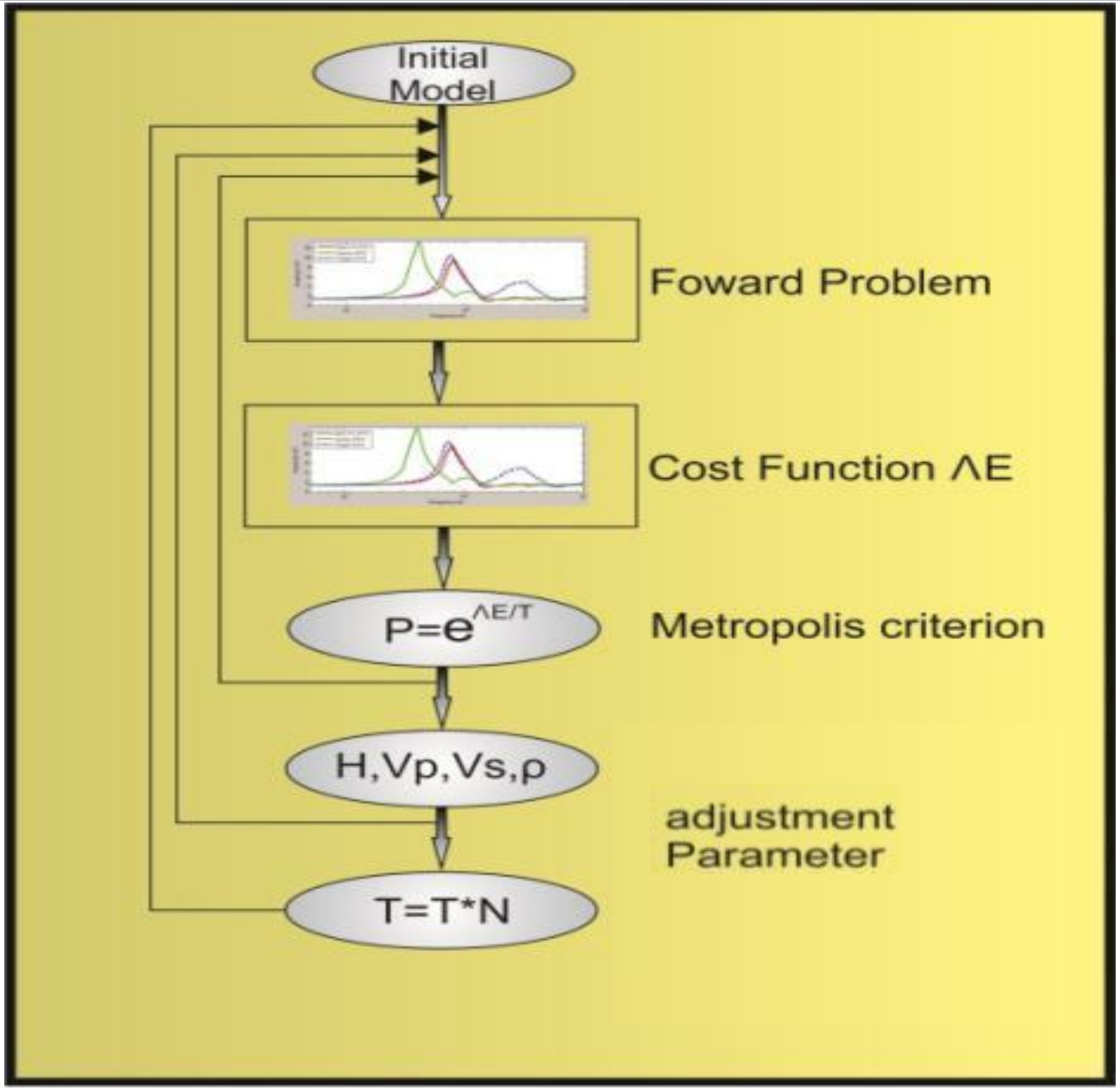


Inverse
Problem

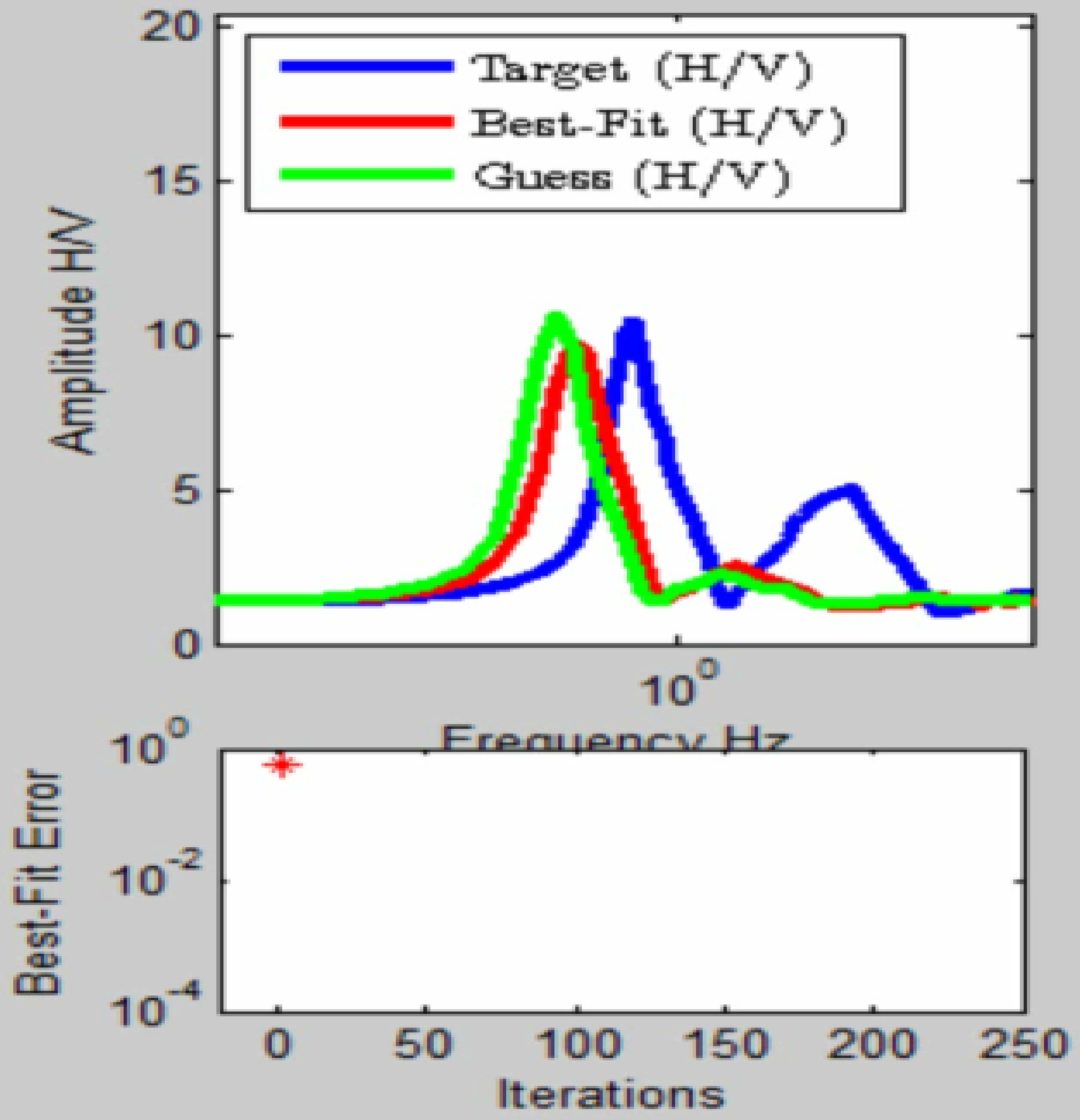


$$\Delta E = \frac{1}{n}\sum_{i}\frac{\left(\left[H/V\left(\omega_i\right)\right]^{Th.} - \left[H/V\left(\omega_i\right)\right]^{Exp.}\right)^2}{\sigma\left(\omega_i\right)^2}$$

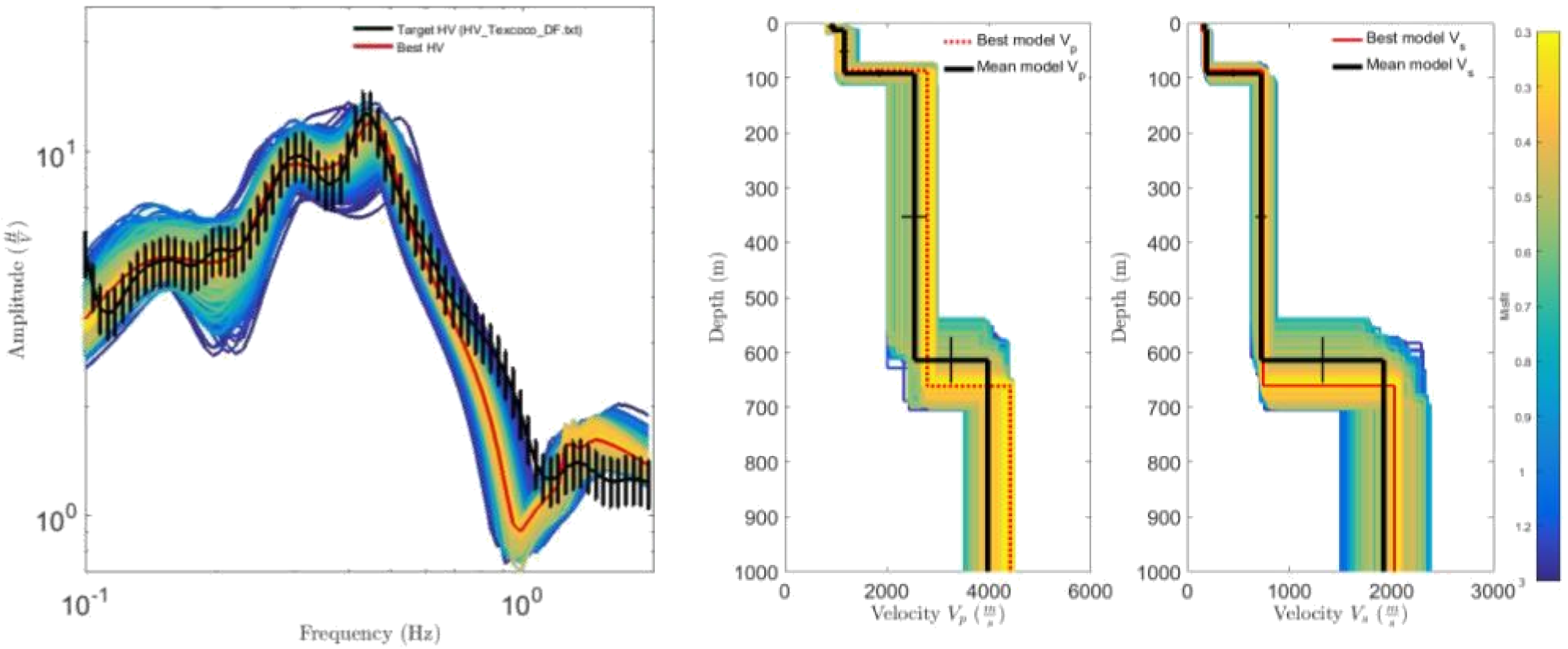
Inversion (Simulated Annealing)



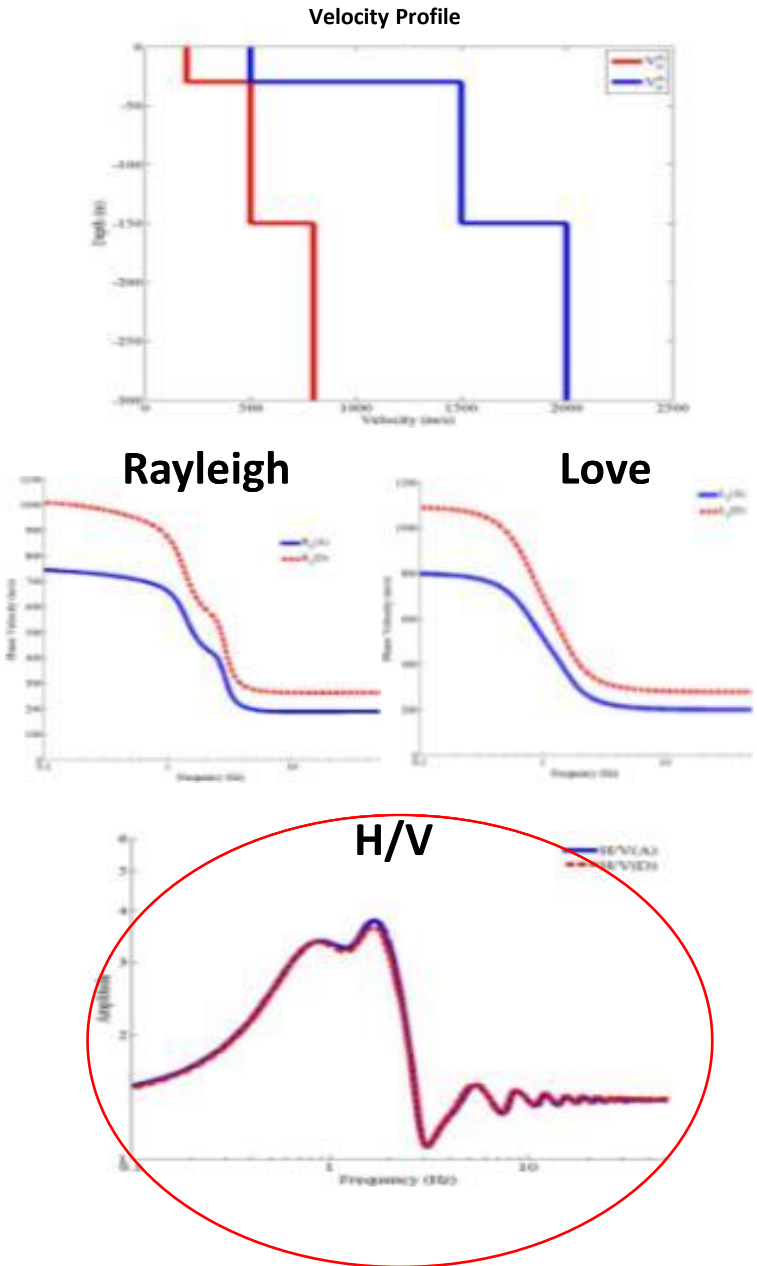
Inversion example of H/V

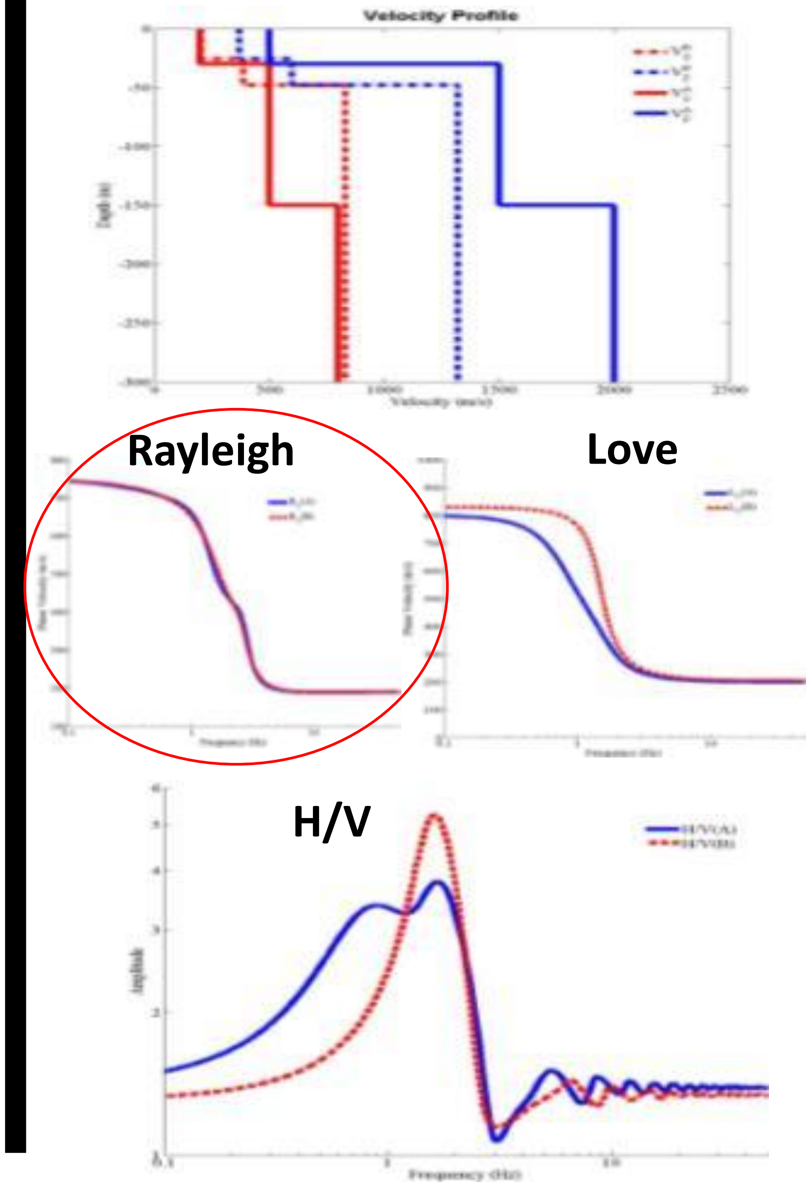


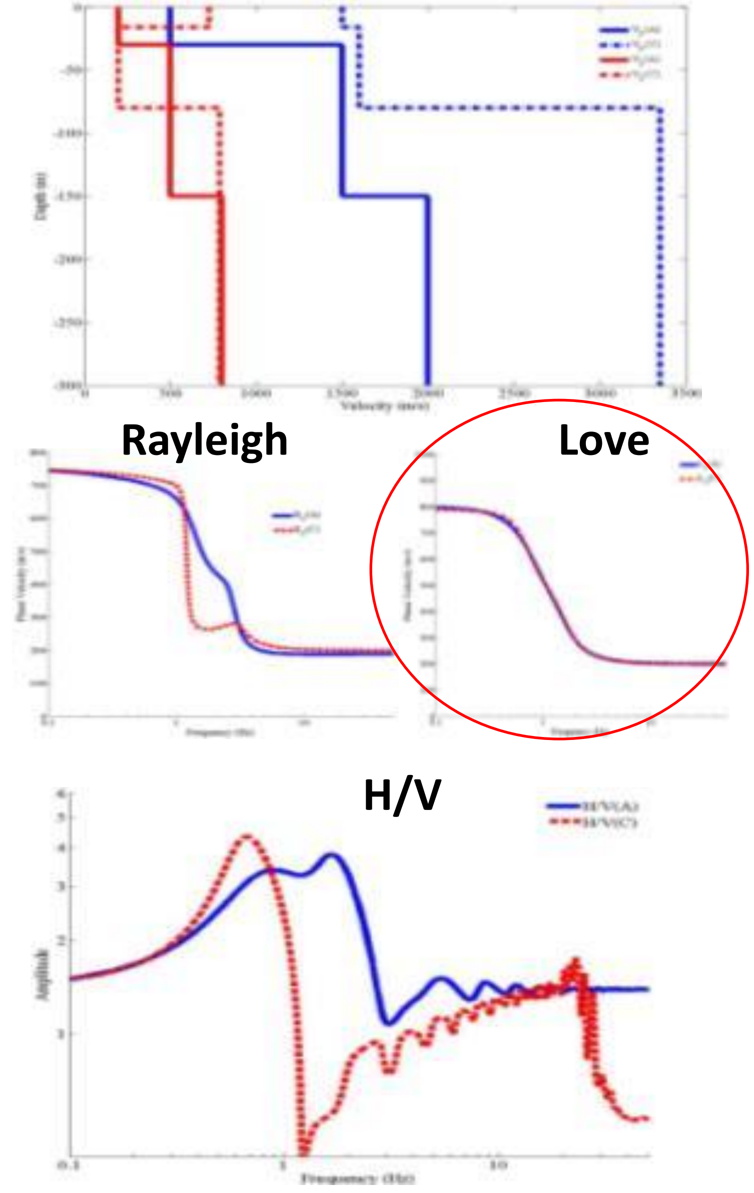
Application to site effect characterization at Texcoco, México D.F.



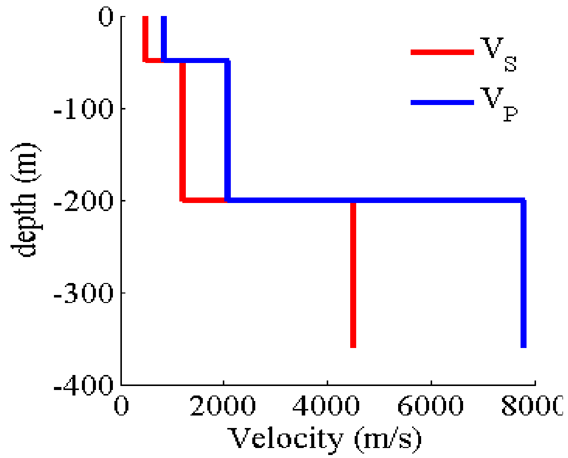
Non-uniqueness of H/V, dispersion curves

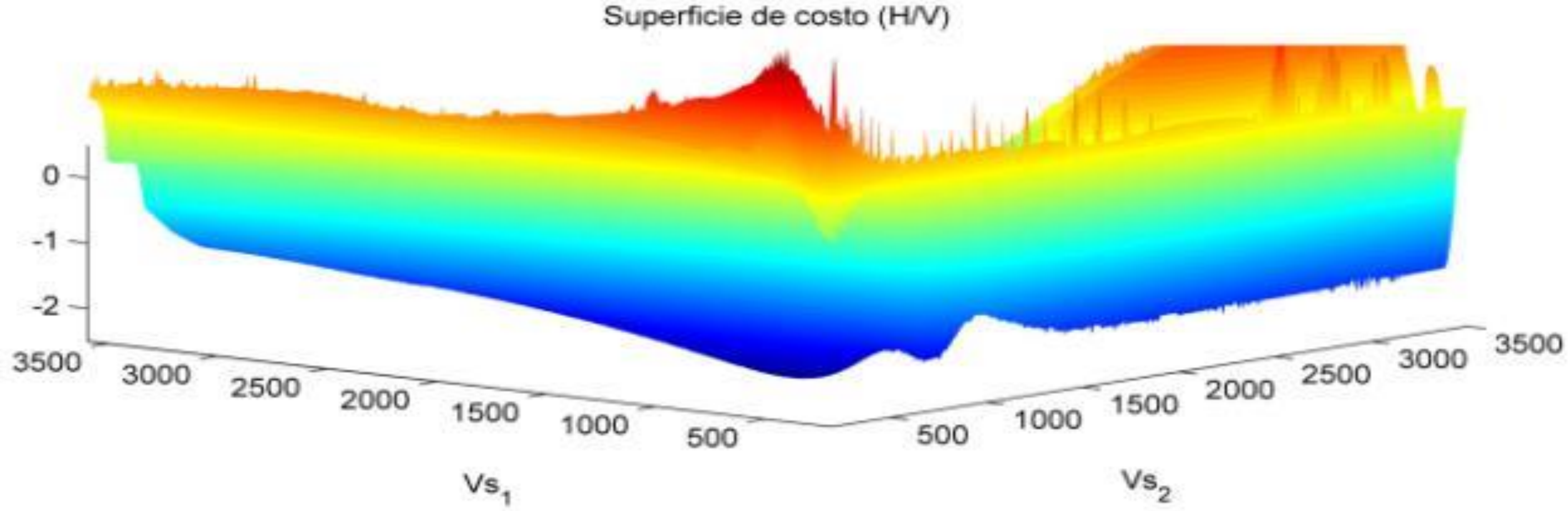


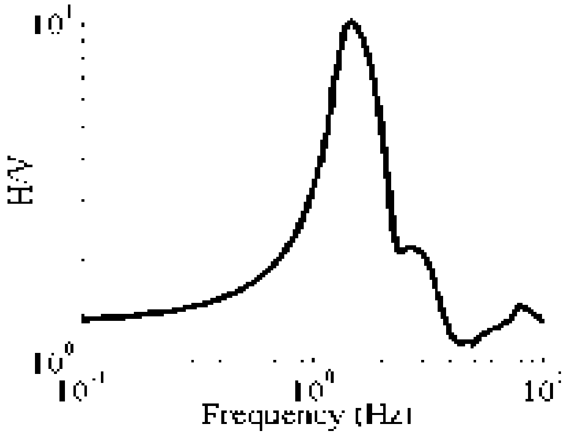


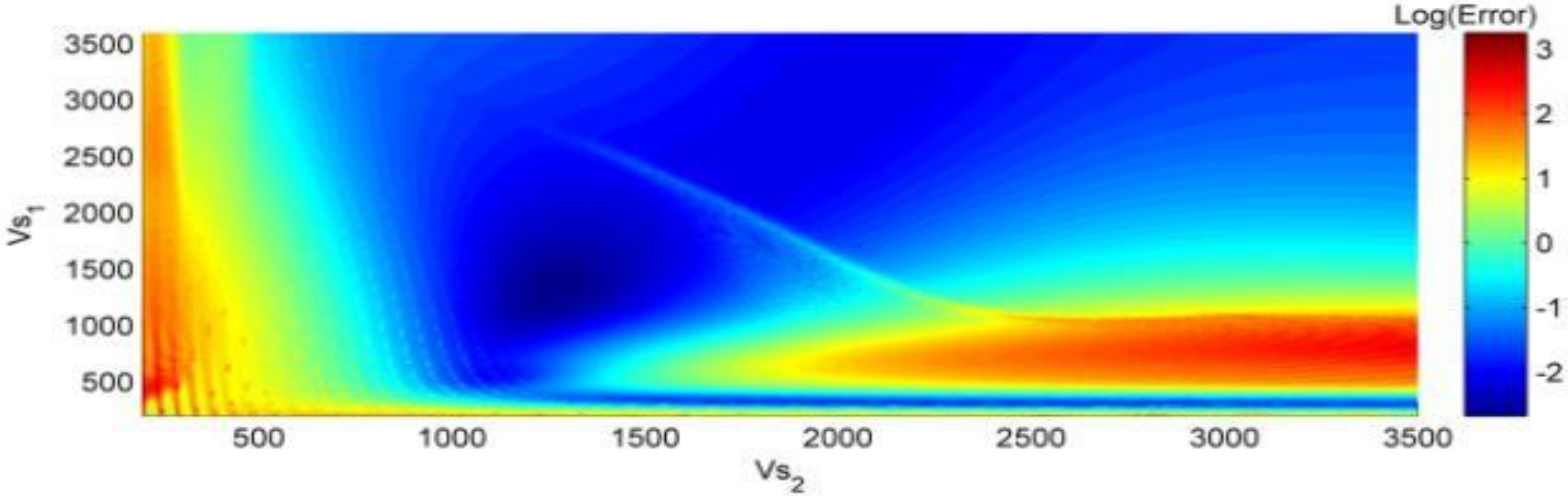


Cost function map for H/V & dispersion curves joint inversion

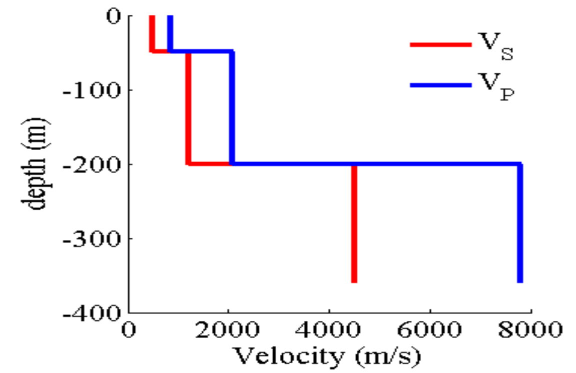


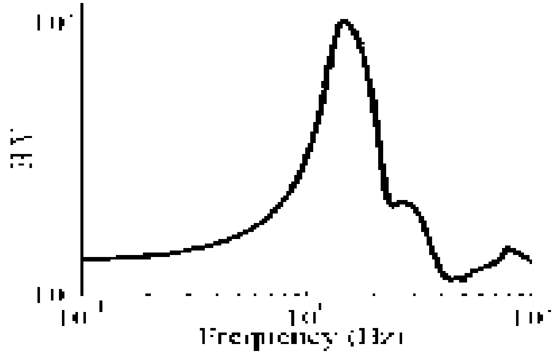


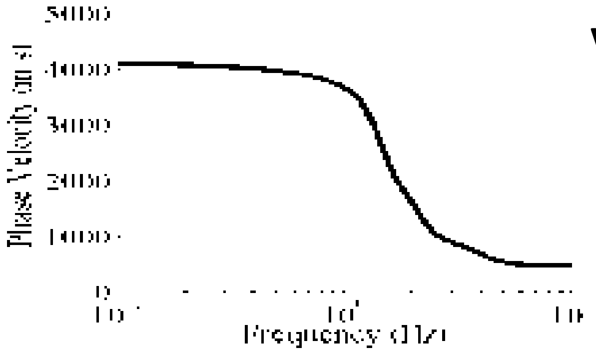


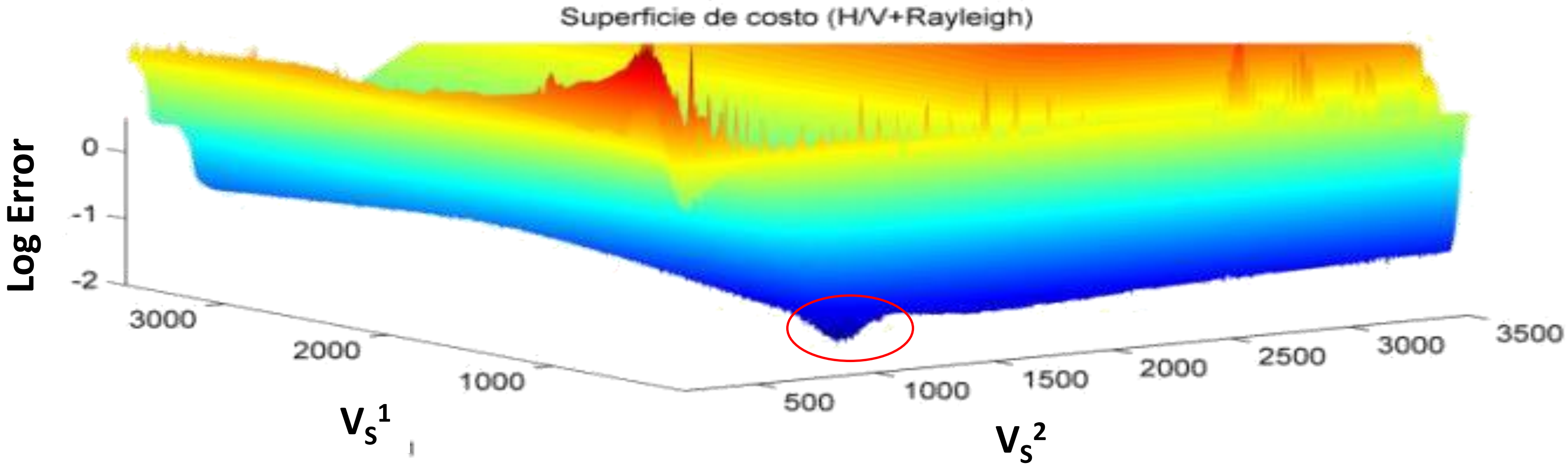


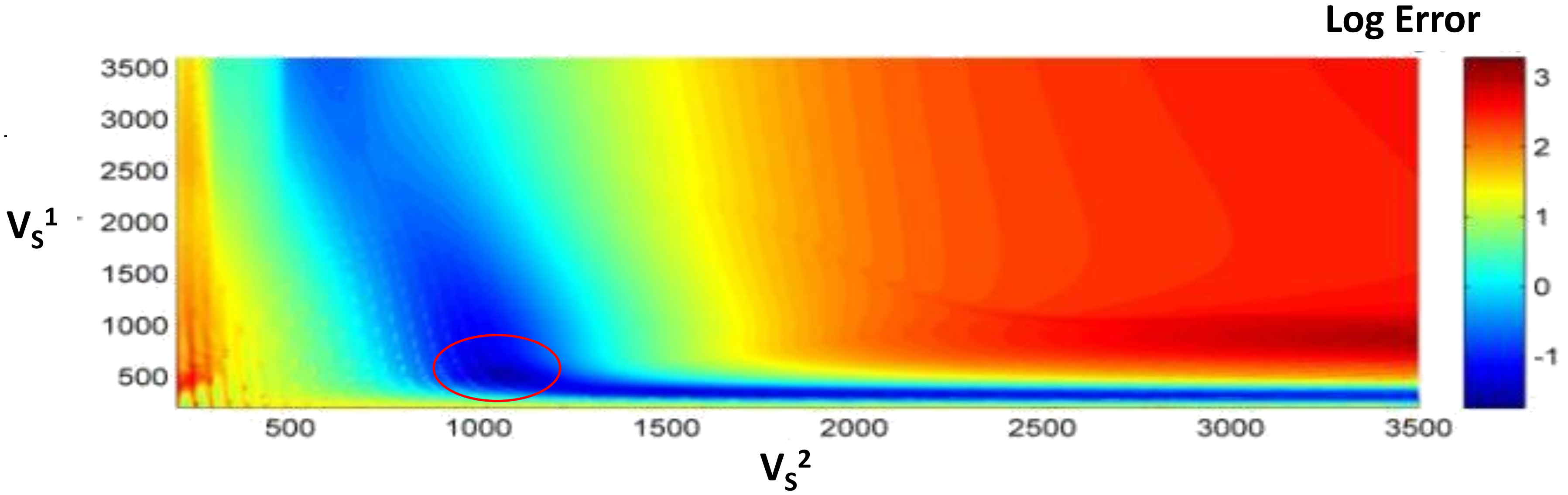
Cost function map for H/V & dispersion curves joint inversion



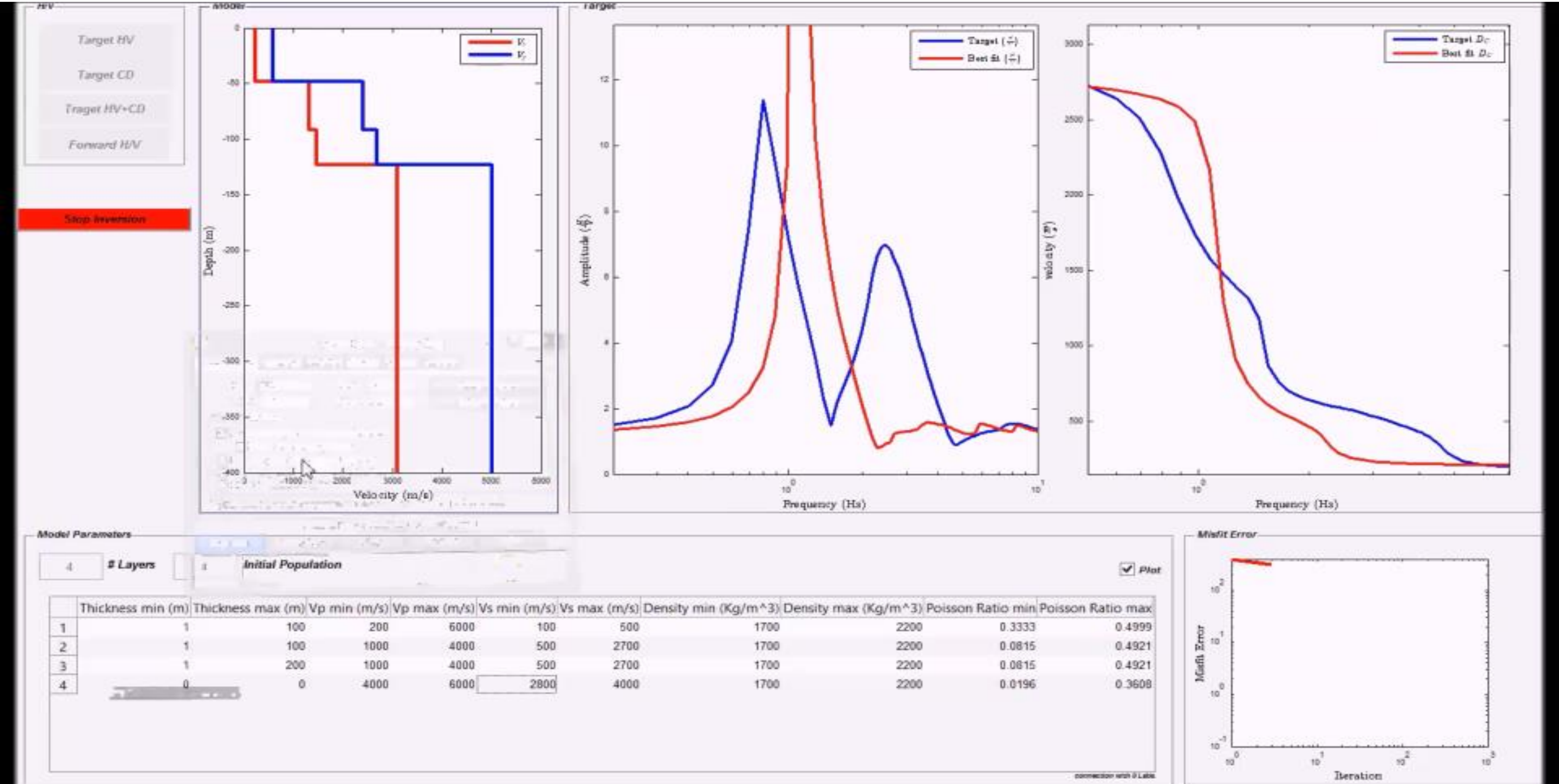






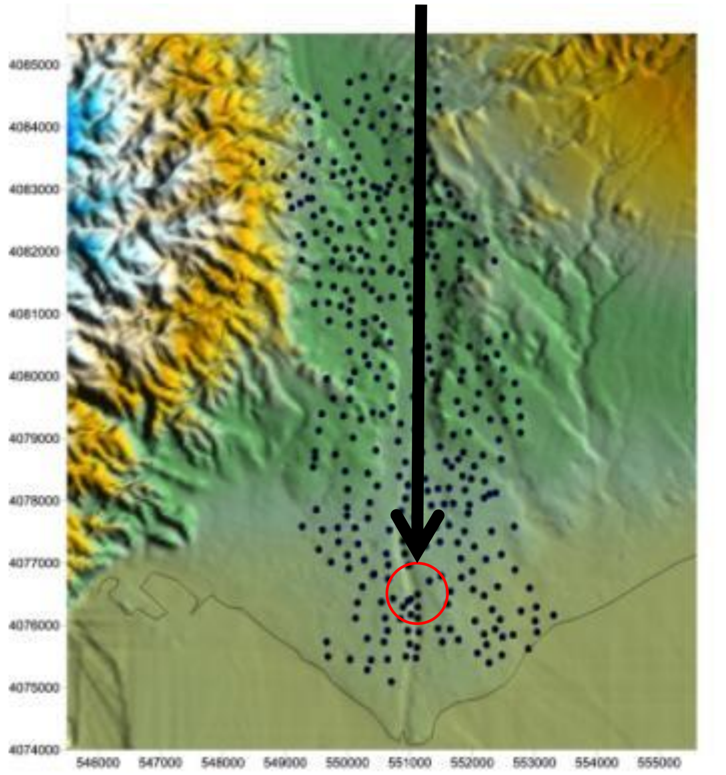


Example of joint inversion



Application to site effect characterization at Almería, Río Andarax, Spain (1/2)

SPAC - Pentagonal Array Rmax 450m

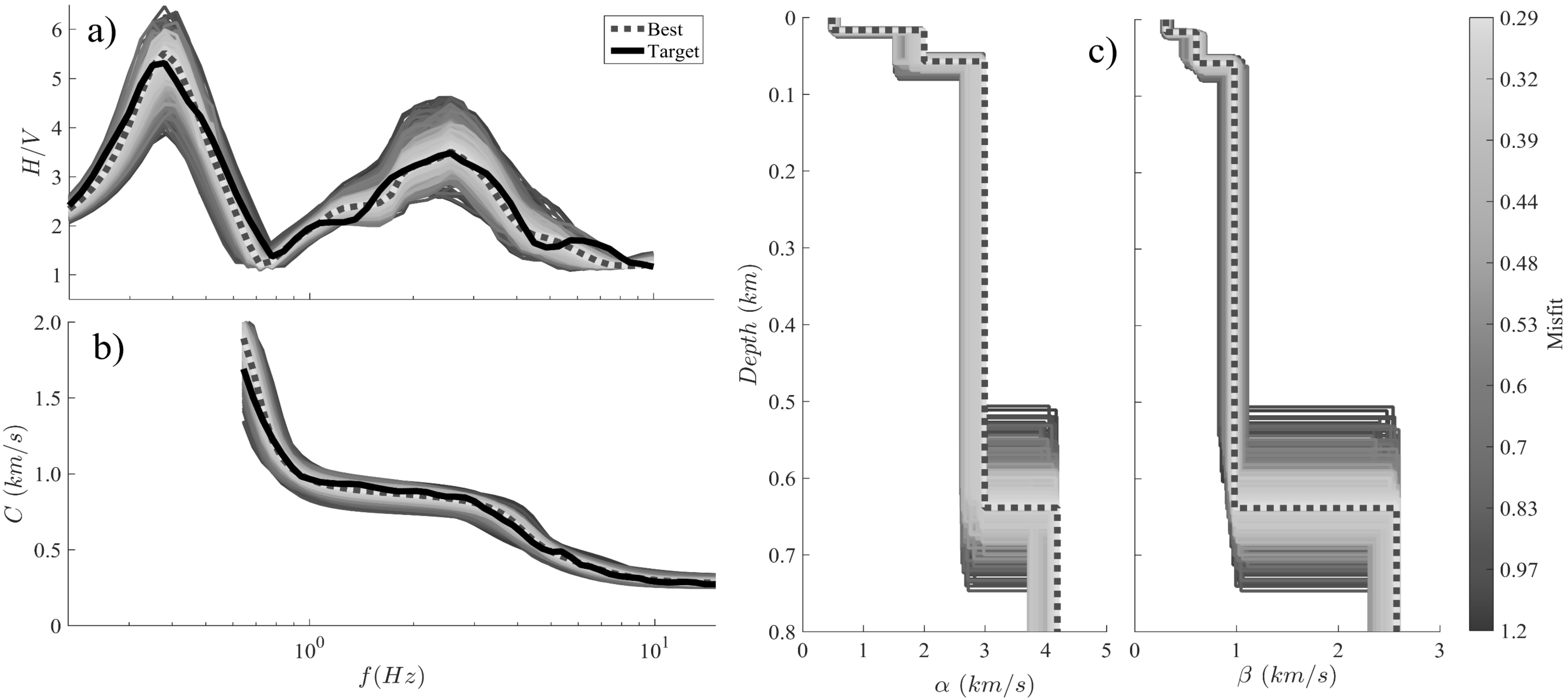


Stations distribution
(from Dr. E. Carmona)

At each station, we computed H/V

Also local dispersion curves are computed using SPAC technique

Application to site effect characterization at Almería, Río Andarax, Spain (2/2)



Conclusions

- Green's function (GF) can be retrieved from correlations within a diffuse field.
- Directional Energy Densities from autocorrelations are related with GF.

$$E_1(x,\omega) \sim \langle |u_1(x,\omega)|^2 \rangle \sim \mathrm{Im}G_{11}(x,x,\omega).$$

- Assuming noise is diffusive:

$$\frac{H}{V}(x,\omega) = \sqrt{\frac{E_1(x,\omega)+E_2(x,\omega)}{E_3(x,\omega)}} = \sqrt{\frac{\mathrm{Im}G_{11}(x,x,\omega)+\mathrm{Im}G_{22}(x,x,\omega)}{\mathrm{Im}G_{33}(x,x,\omega)}}$$

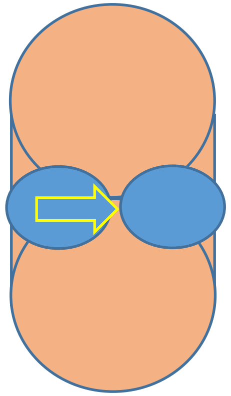
- This expression relates Field Measurements and System's Properties, and allows extraction of soil information hidden in ambient seismic noise. Even if noise is not fully diffusive, residual coherency may allow to retrieve Green's functions.
- Appropriate data processing H/V may allow the inversion of soil profile and then its effects in strong ground motion can be explored.

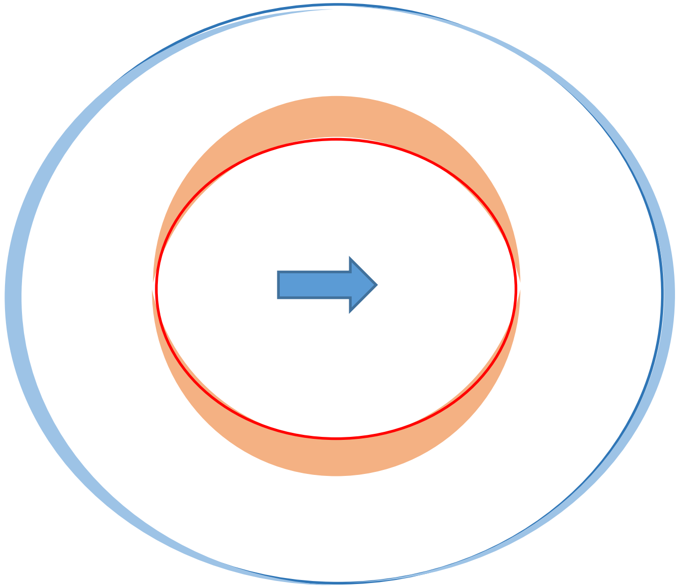
Two References

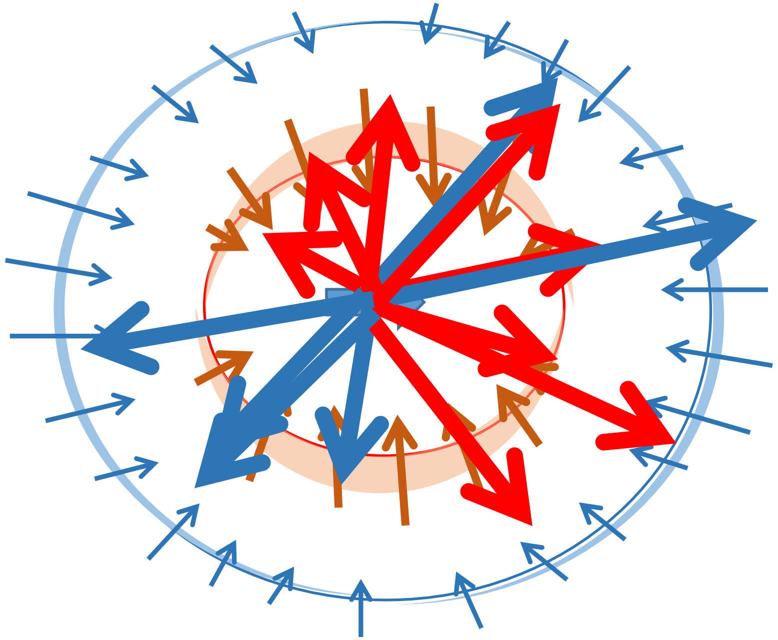
A García-Jerez *et al*. (2016), A computer code for forward calculation and inversion of the H/V spectral ratio under the diffuse field assumption, *Computers and Geosciences*, in press.

J Piña-Flores *et al*. (2016), The inversion of spectral ratio H/V in a layered media using the diffuse field assumption, *Geophysical Journal International*, Submitted.

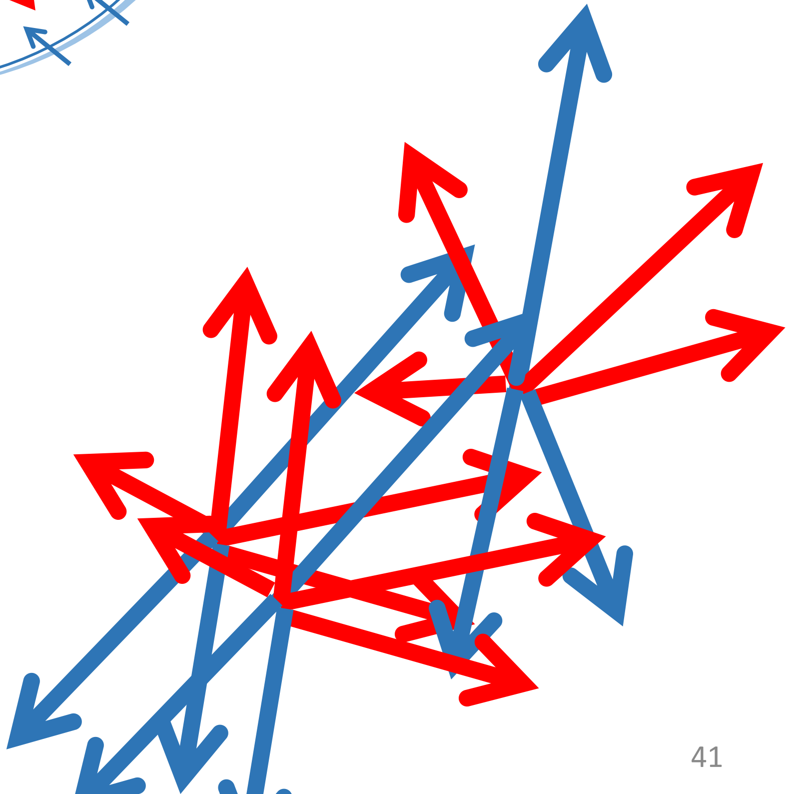
Thank you !

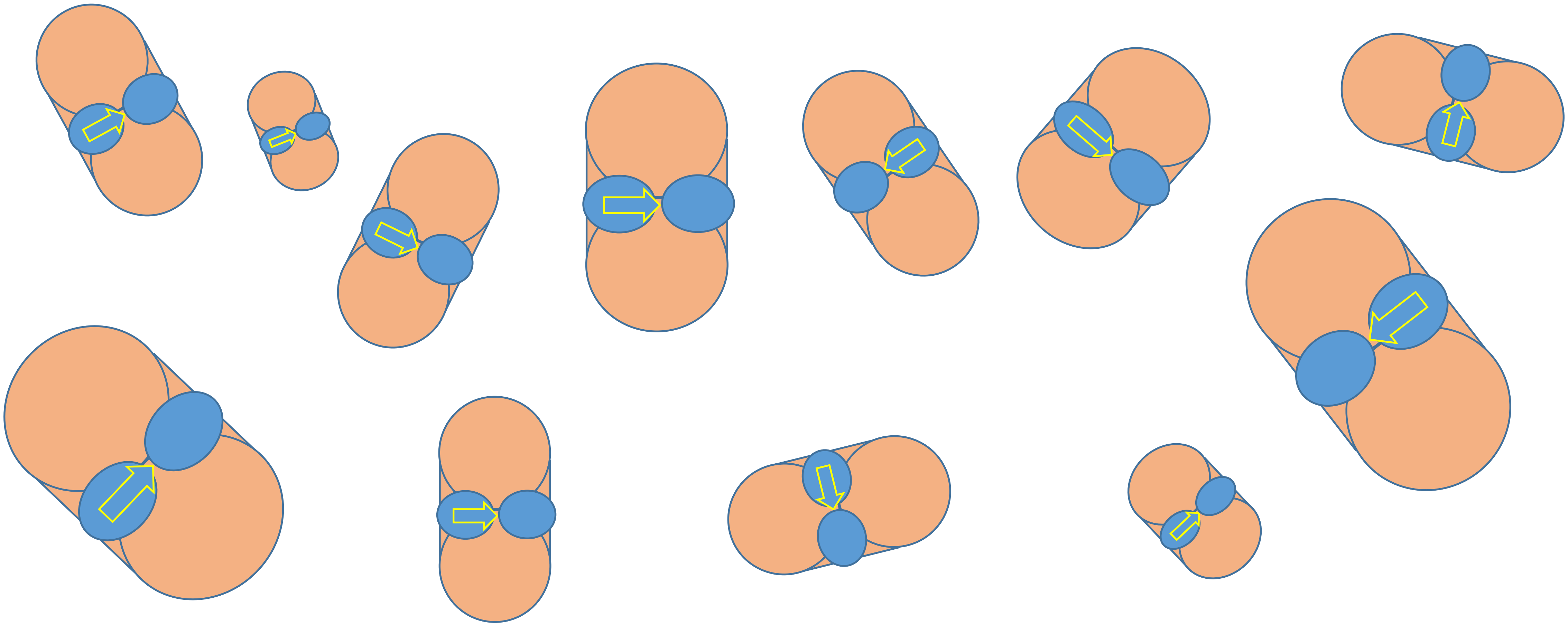


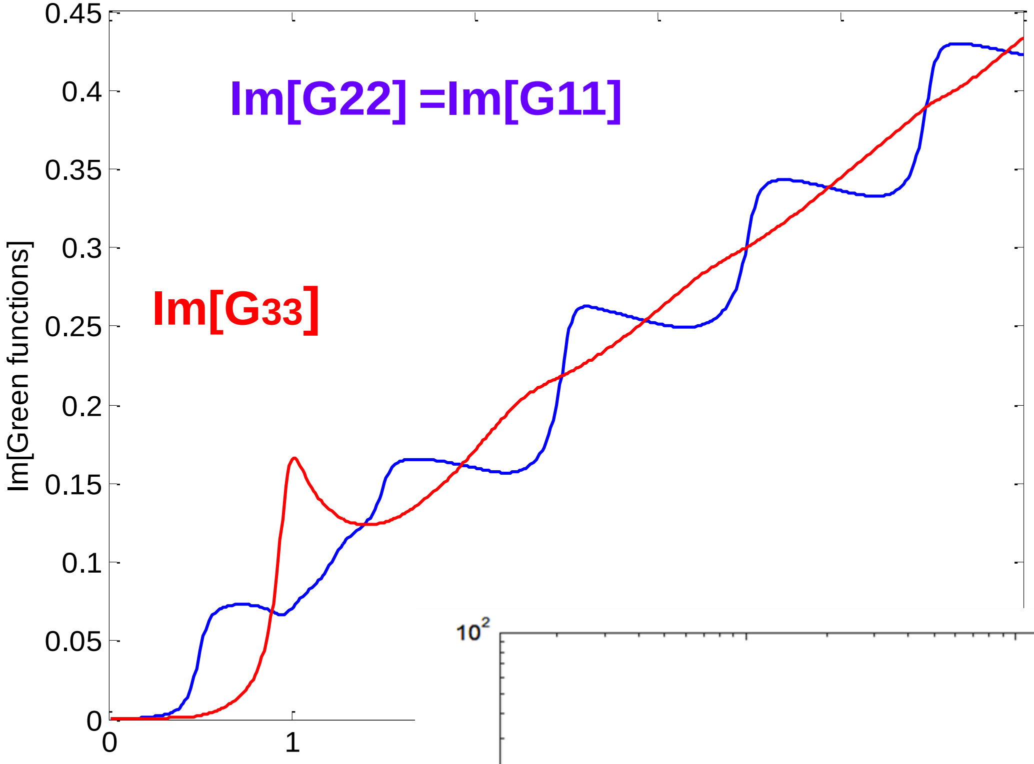


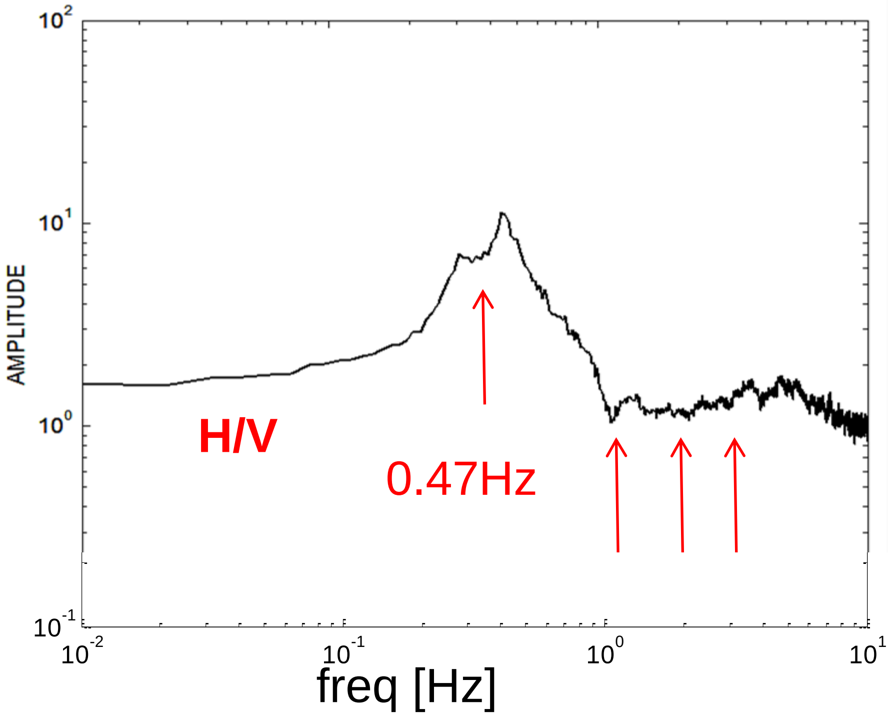


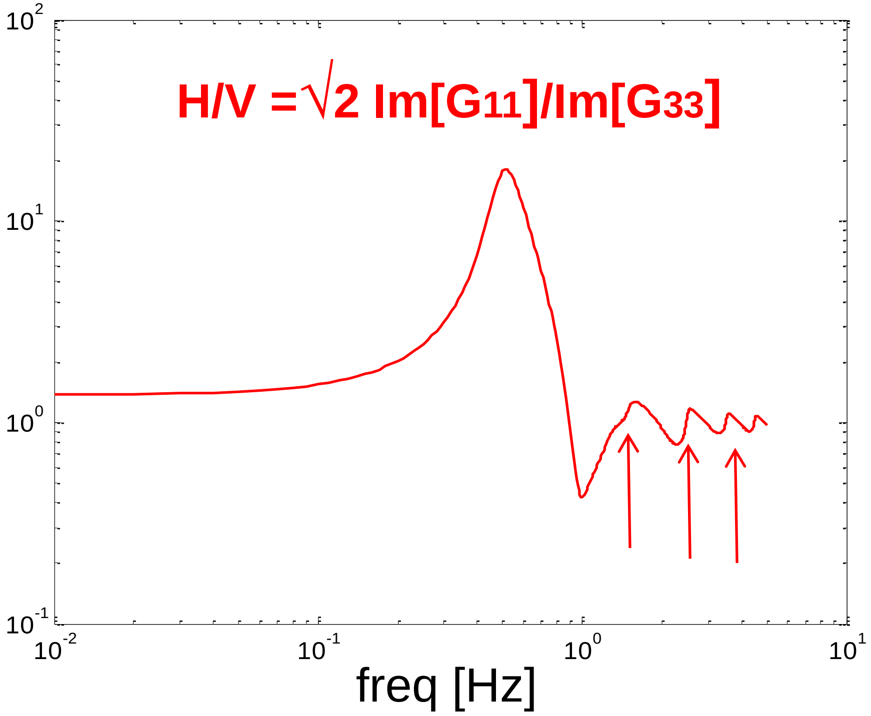
$$\mathrm{Im}[G_{11}(\mathbf{x},\mathbf{x},\omega)] = \frac{-\omega}{12\pi\rho} \times \left\{ \frac{2}{\beta^3} + \frac{1}{\alpha^3} \right\}$$

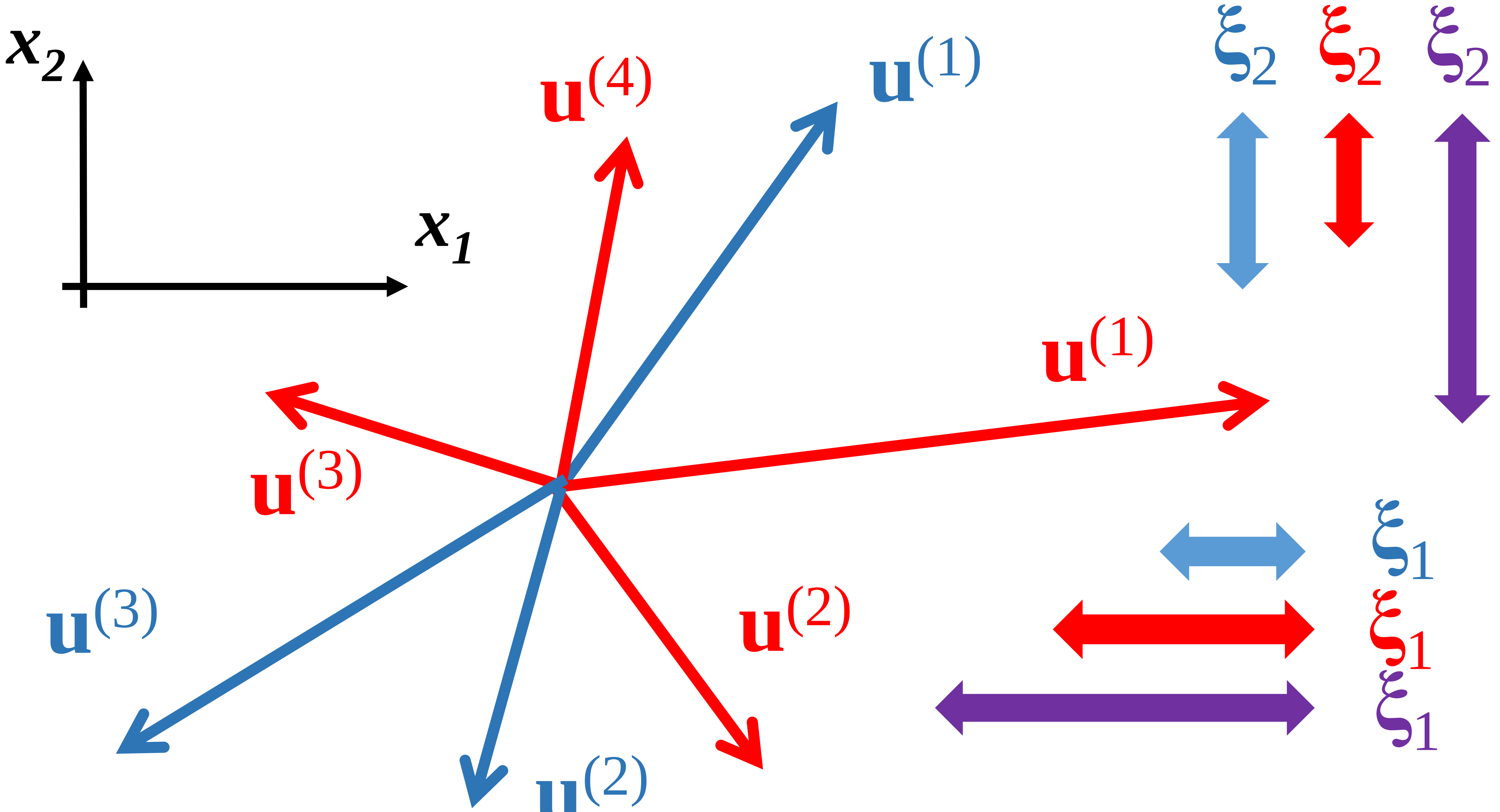




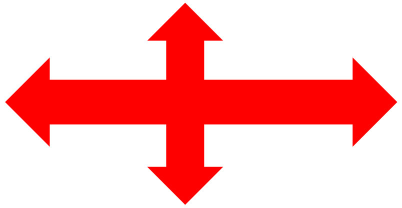

Im[G22] =Im[G11]
Im[G33]
Im[Green functions]
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1

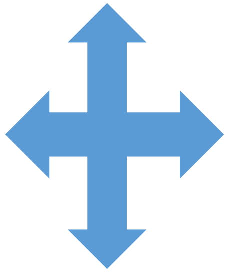

AMPLITUDE
H/V
0.47Hz
freq [Hz]

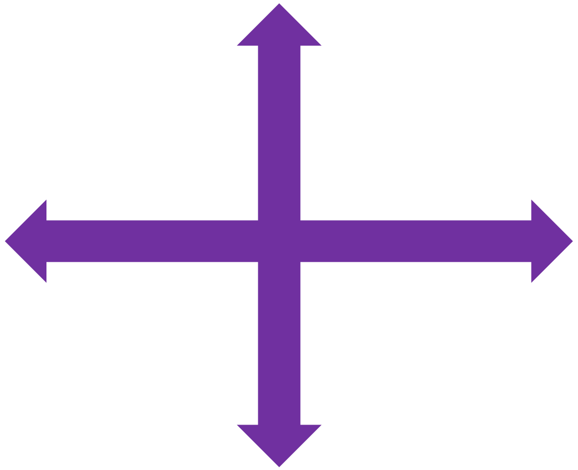

H/V =√2 Im[G11]/Im[G33]
freq [Hz]


x_2
x_1
$\mathbf{u}^{(4)}$
$\mathbf{u}^{(1)}$
ξ_2 ξ_2 ξ_2
$\mathbf{u}^{(1)}$
$\mathbf{u}^{(3)}$
ξ_1
$\mathbf{u}^{(3)}$
$\mathbf{u}^{(2)}$
ξ_1
ξ_1
$\mathbf{u}^{(2)}$

$$\xi_1 + \xi_2 = \xi$$







$$\xi_1 + \xi_2 = \xi$$

$$\xi + \xi = \xi$$