Two-dimensional wavefield reconstruction: Tsunami data assimilation and seismic gradiometry

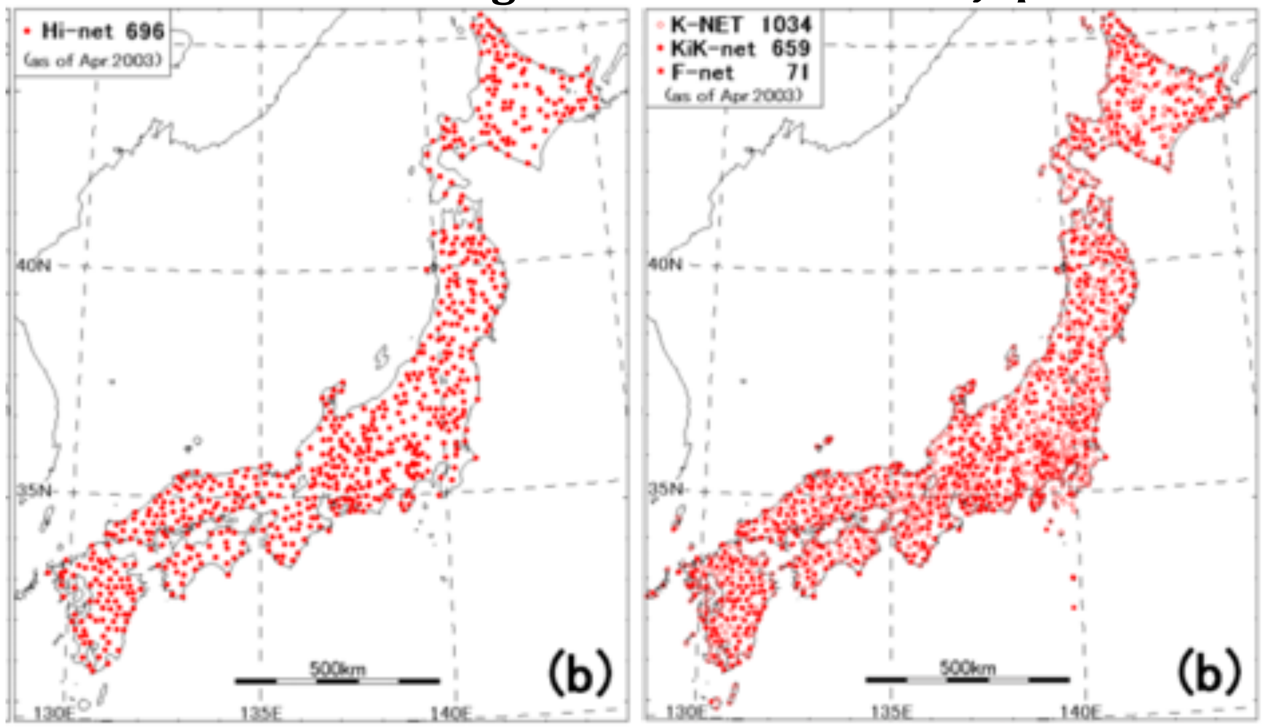
Takuto Maeda

Earthquake Research Institute

The University of Tokyo

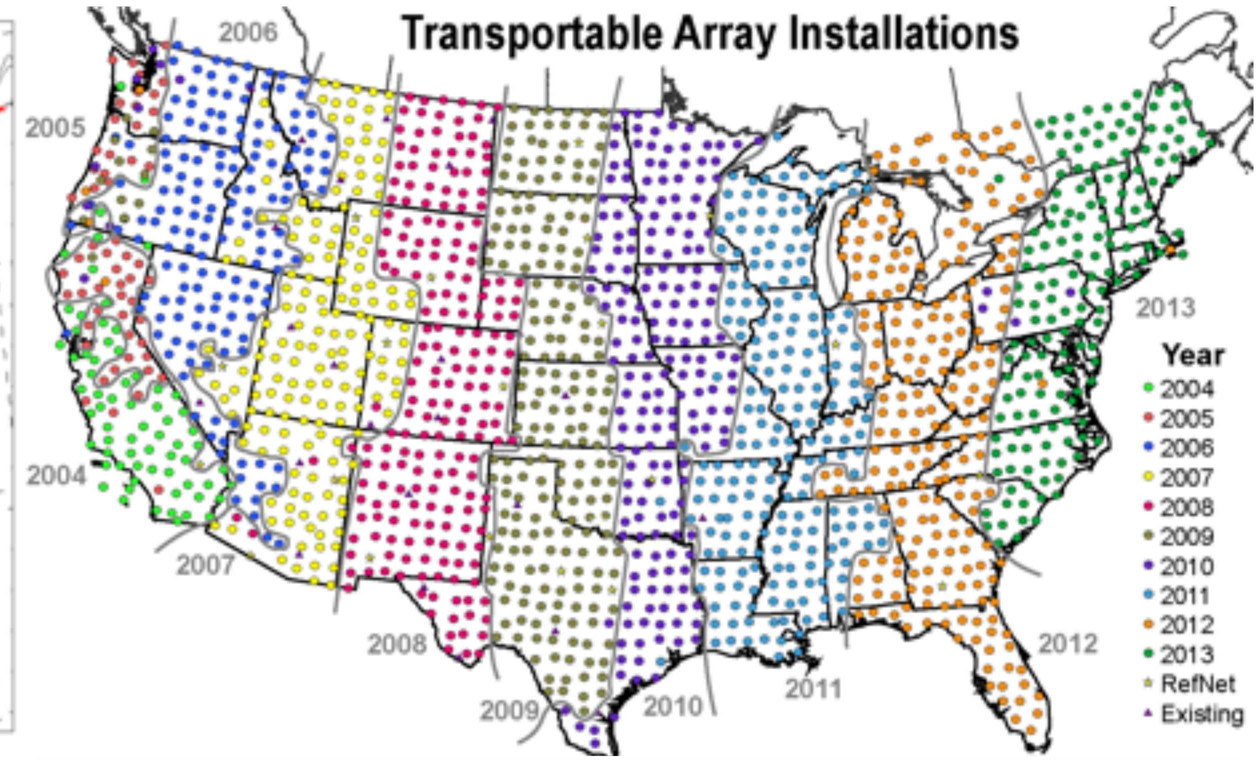
The “Large-*N*” arrays all over the world

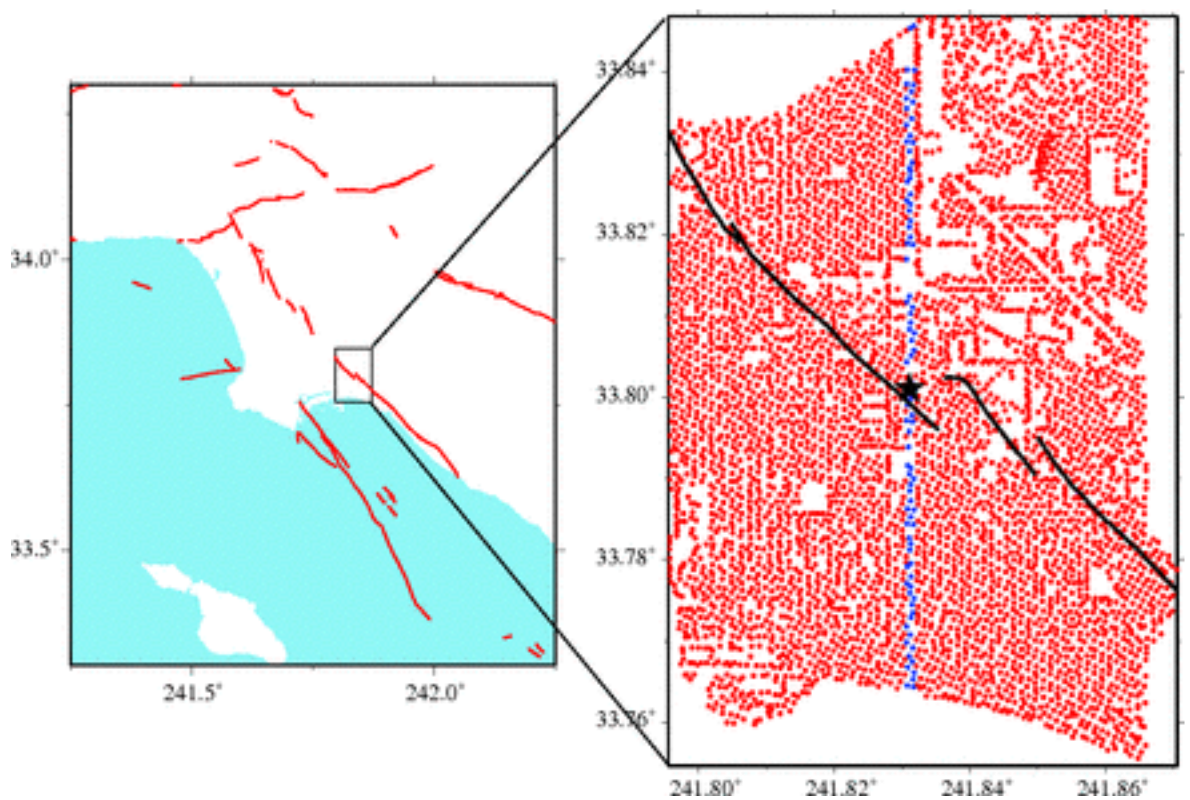
Hi-net & Strong Motion Networks in Japan



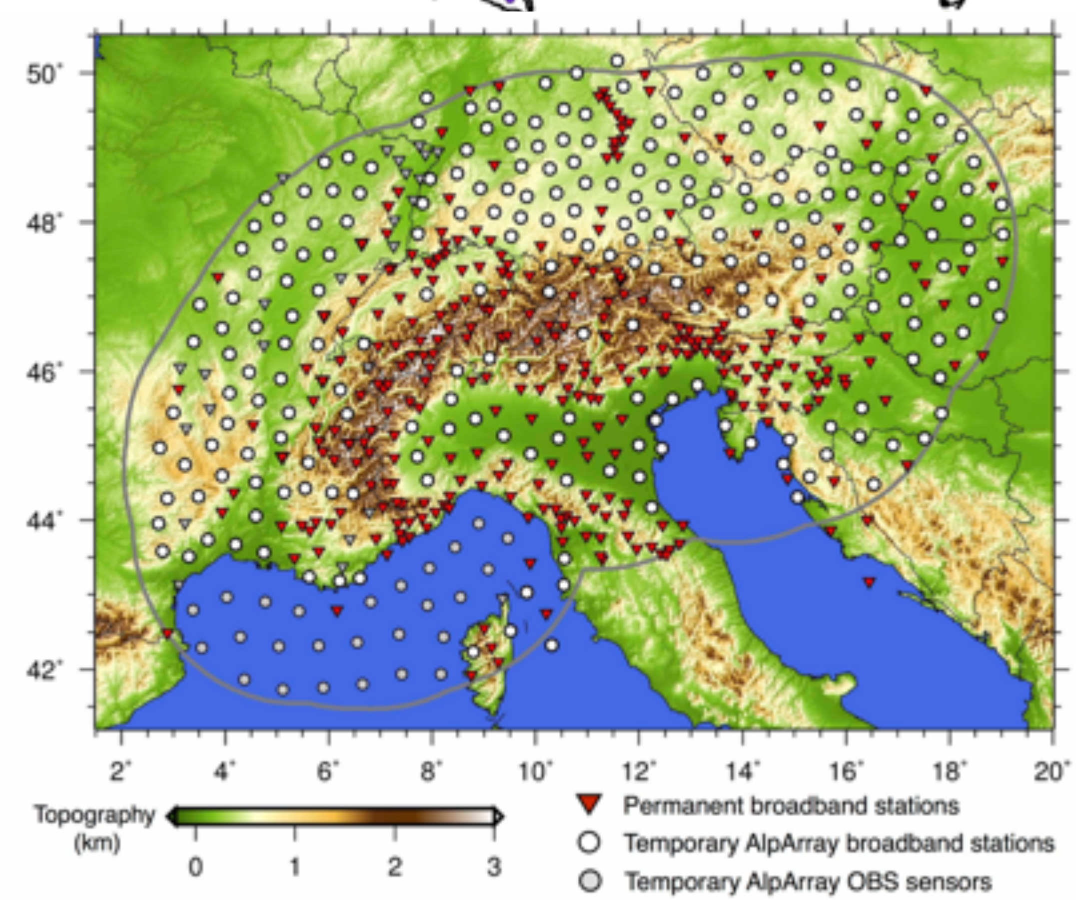
(Okada et al., 2004)

USArray



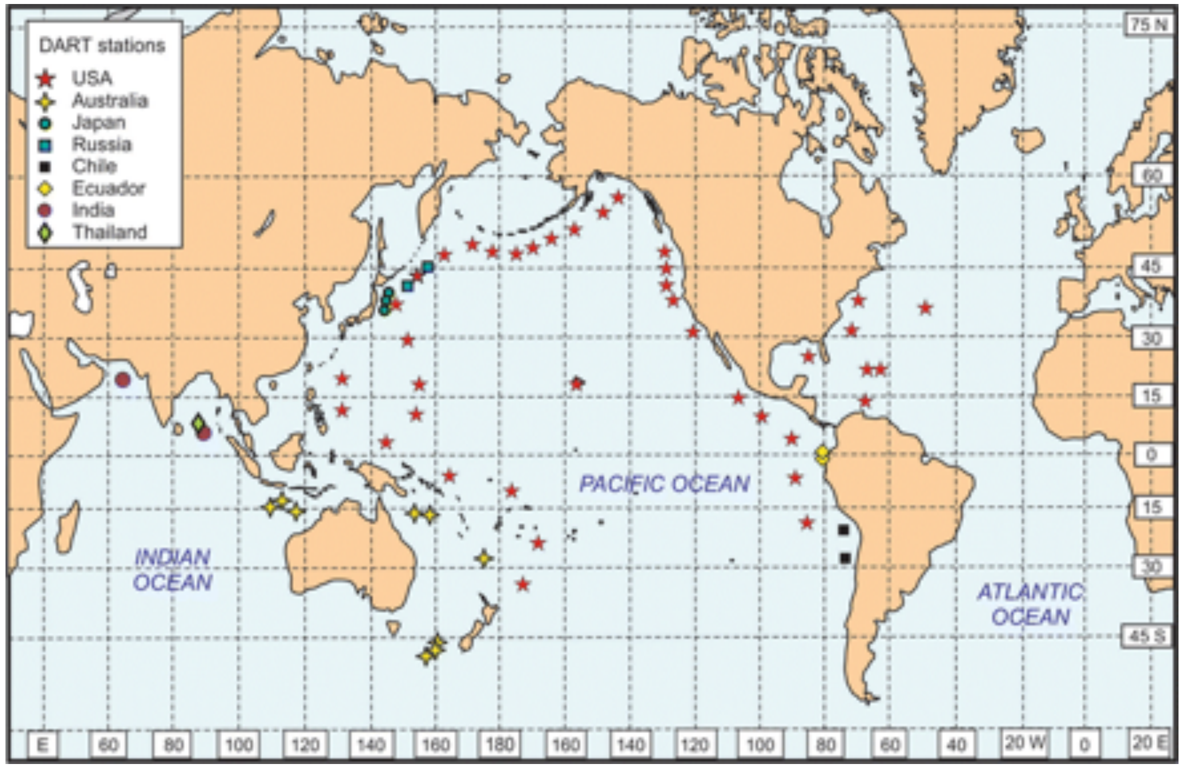


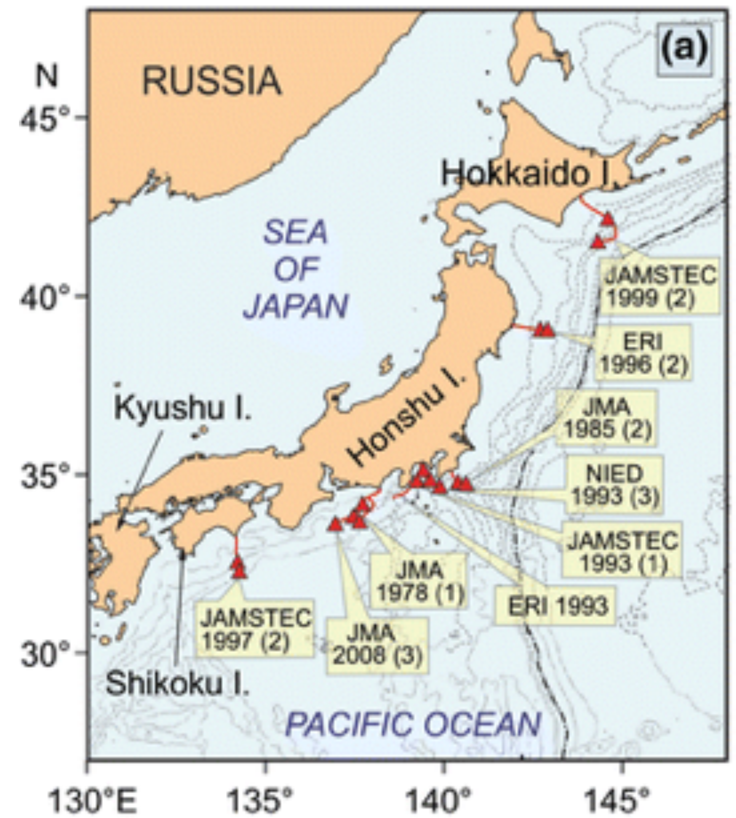
Long Beach Array (Lin et al., 2012)

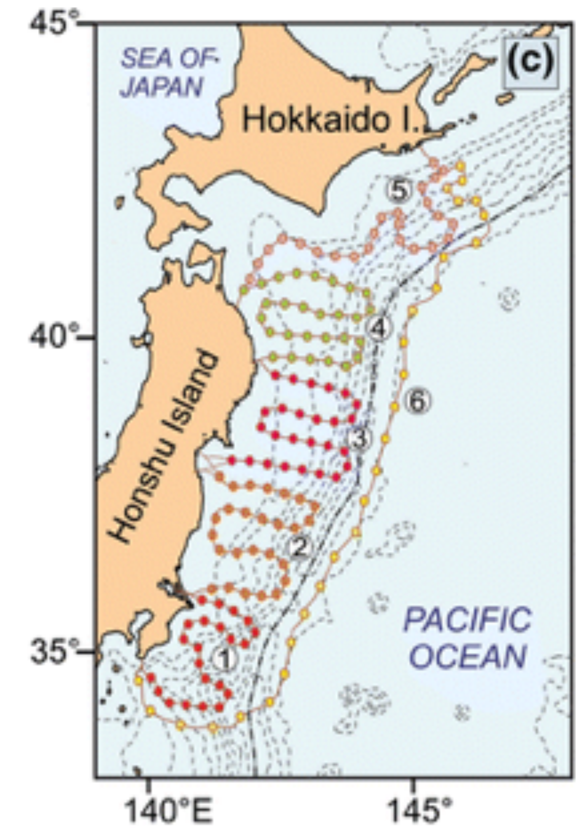


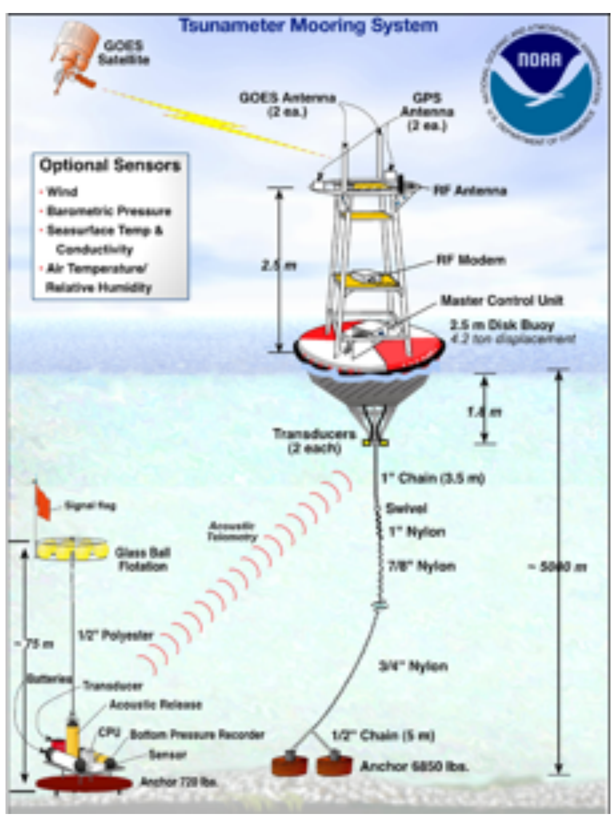
The AlpArray Initiative (http://www.alparray.ethz.ch)

Development of dense tsunami networks



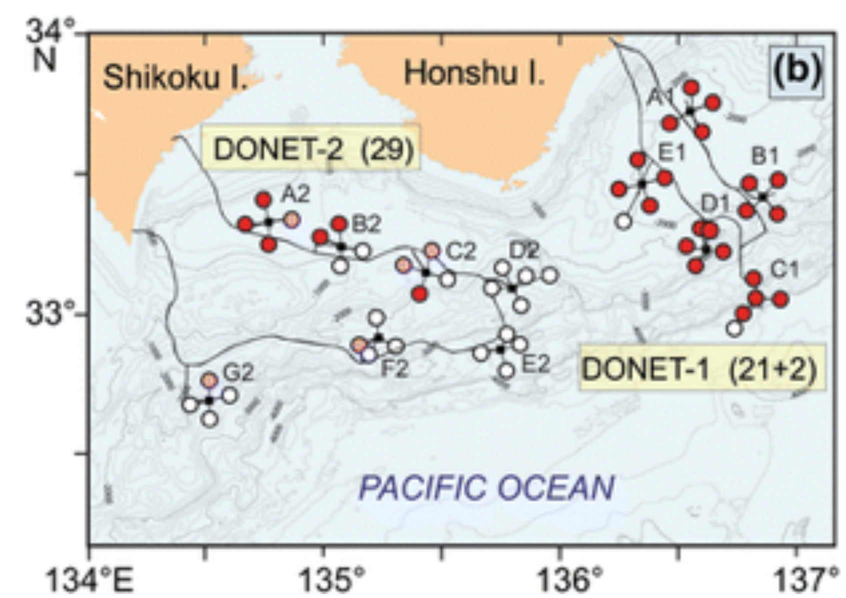






(noaa.gov)

(www.bosai.go.jp)



(Rabinovich and Eble, 2015, PAGEOPH)

Motivation

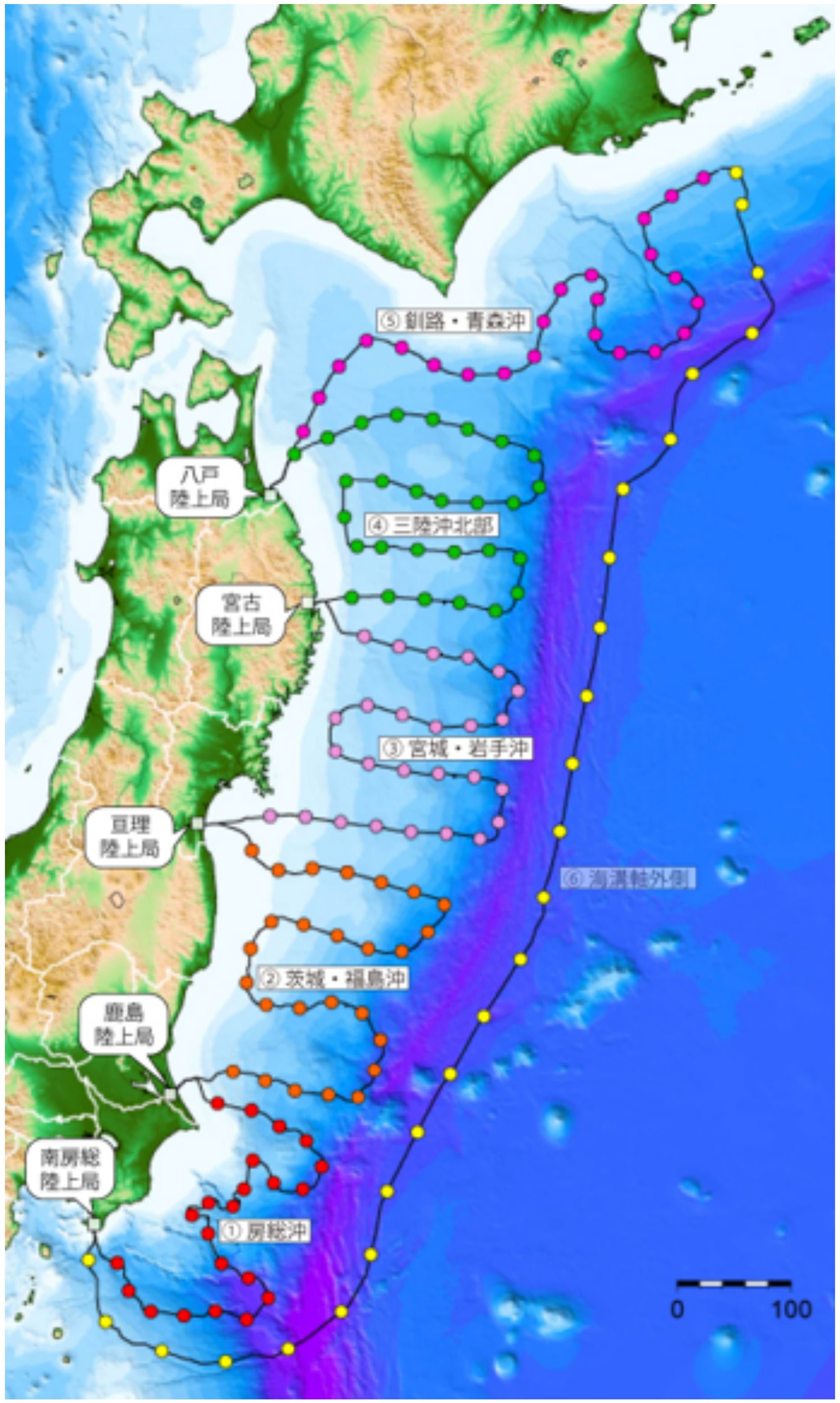
- How to utilize these large dataset ?

  - for understandings inhomogeneous subsurface structure
    - for more deep understandings of physics of wave propagation in heterogeneous media

  - for real-world application, in particular early warnings of earthquakes and tsunamis

- Significant improvement on station density compared to wavelength
  - Obtain more information through two-dimensional continuous wavefield modeling
  - Independent two topics on tsunami and seismic wave propagation

The new *S-net*



(www.bosai.go.jp)

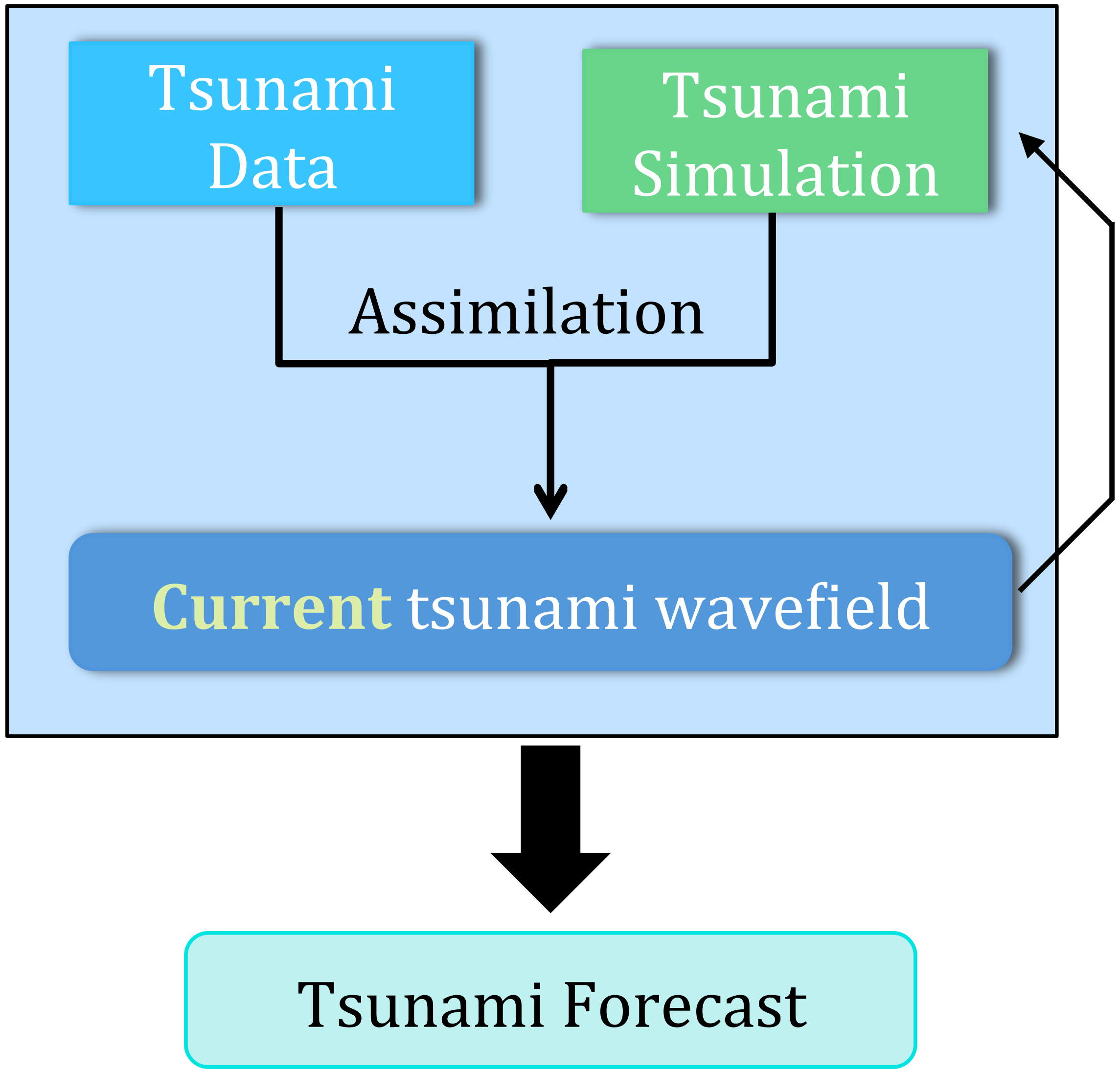
- Super dense realtime network for seismic & tsunami wave monitoring
- A part of the network started observation from 2016

- With a dense network, we may be able to track **2D tsunami wavefield**
- No source is necessary for forecasting ?

A new approach: *Data assimilation*

- **Not relying on source data**
- **Estimate wavefield**

- Directly fit tsunami simulation with observation
- Estimated wavefield is further used for better fit of tsunami at next timestep
- Always running = monitoring
- Tsunami forecast can be done whenever it is necessary



Data assimilation as a feedback system

- #1) Forecast by numerical simulation of linear shallow water

$$\eta_n^F(x,y) \leftarrow \eta_{n-1}^A(x,y) - \left(\frac{\partial M(x,y)}{\partial x} + \frac{\partial N(x,y)}{\partial y}\right)\Delta t$$

$$M_n^F(x,y) \leftarrow M_{n-1}^F(x,y) - gh\frac{\partial \eta(x,y)}{\partial x}\Delta t,\ N_n^F(x,y) \leftarrow N_{n-1}^F(x,y) - gh\frac{\partial \eta(x,y)}{\partial y}\Delta t$$

η: tsunami height, M&N: tsunami flow velocity

- #2) Assimilation: A Feedback from **observation residual**

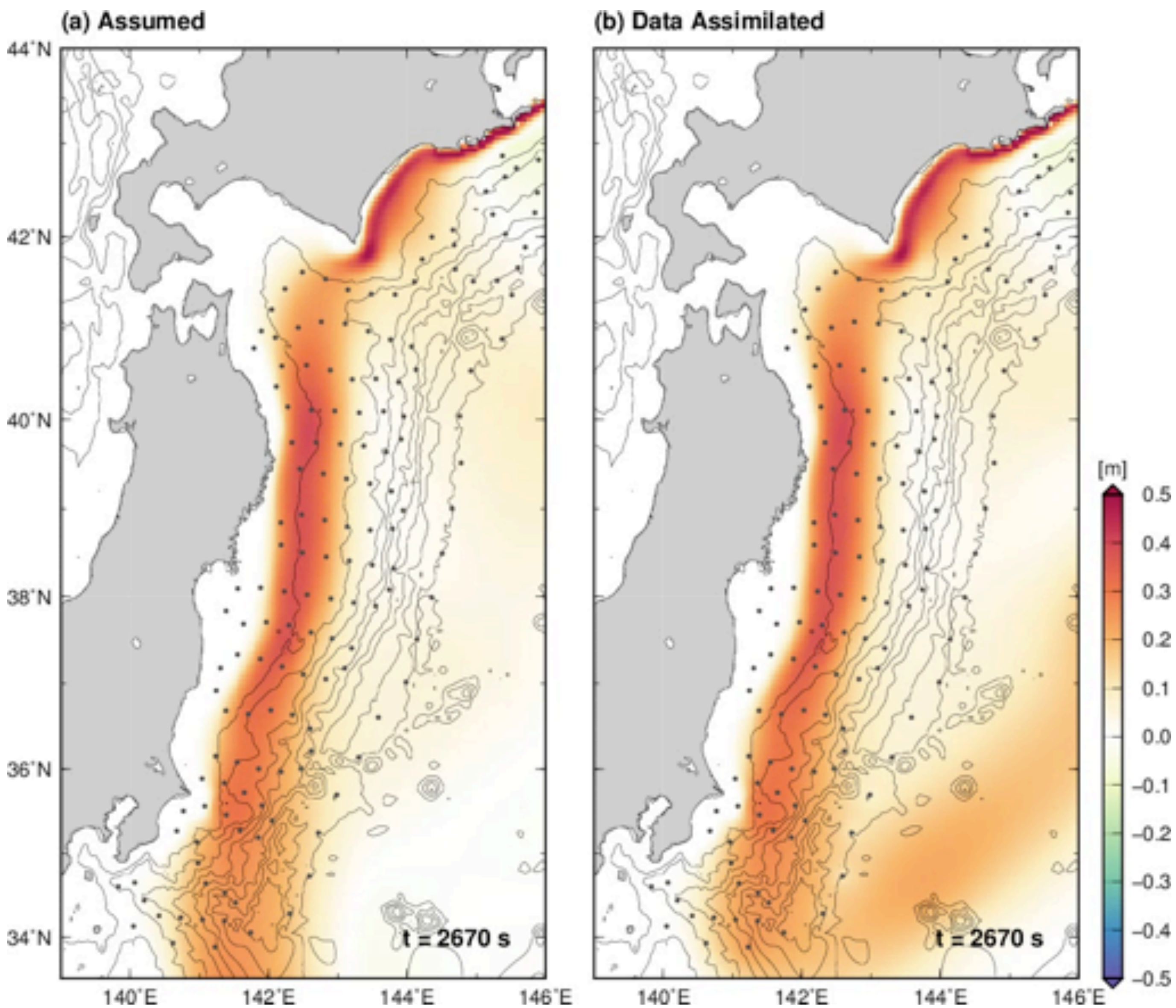
$$\eta_n^A(x,y) \leftarrow \eta_n^F(x,y) + \sum_i W(x,y;x_{S_i},y_{S_i})\left(\eta_n^{\mathrm{obs}}(x_{S_i},y_{S_i}) - \eta_n^F(x_{S_i},y_{S_i})\right)$$

- A weight factor *W* can be estimated by the optimum interpolation algorithm based on the station layout
- The forecasting-assimilation cycle is repeated with updating observed data in real time

(Kalnay, 2003; Maeda et al., 2015)

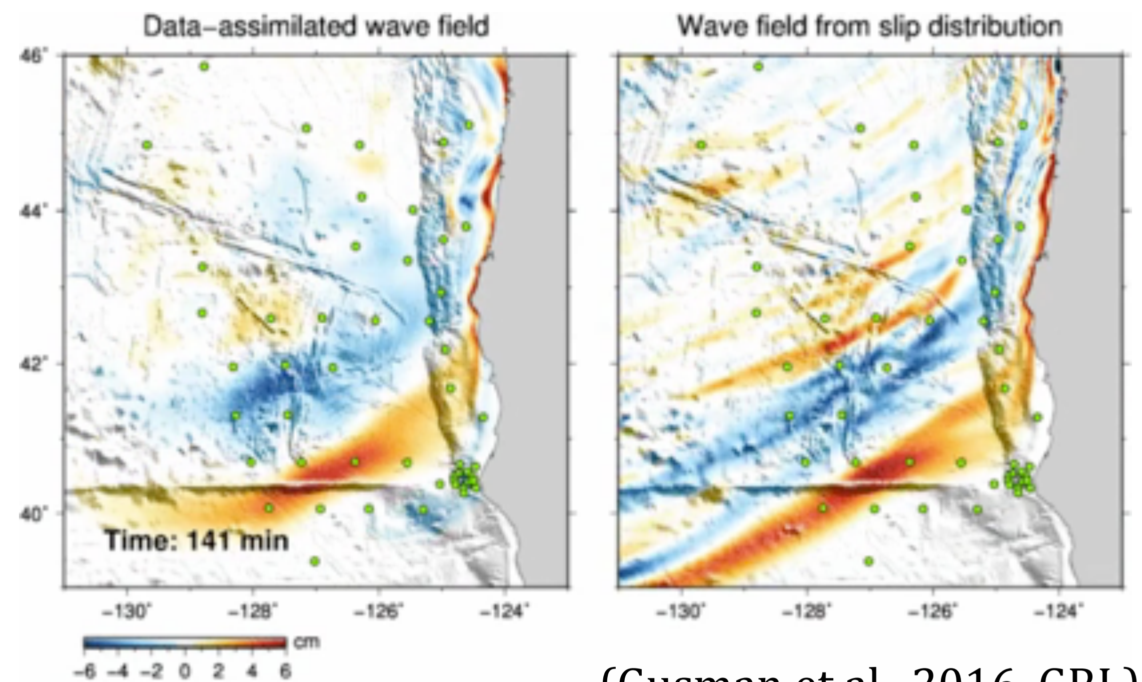
Far-field tsunami forecast by the DA

Numerical forecast experiment



(Maeda et al., 2015, GRL)

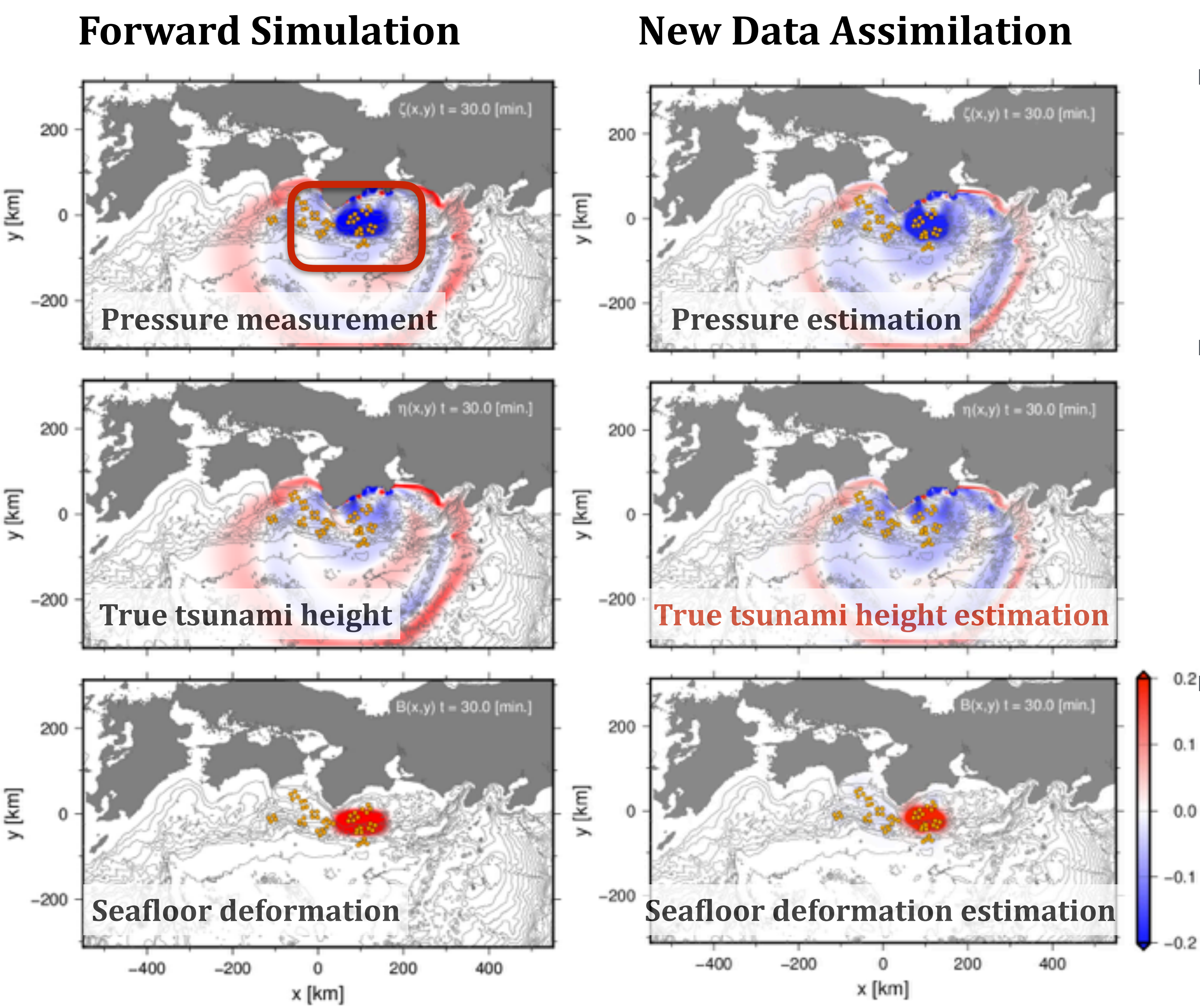
Real-world *post*casting with Cascadia Initiative OBPGs



(Gusman et al., 2016, GRL)

- Reconstruct continuous wavefield through assimilation

Near-field pressure problem

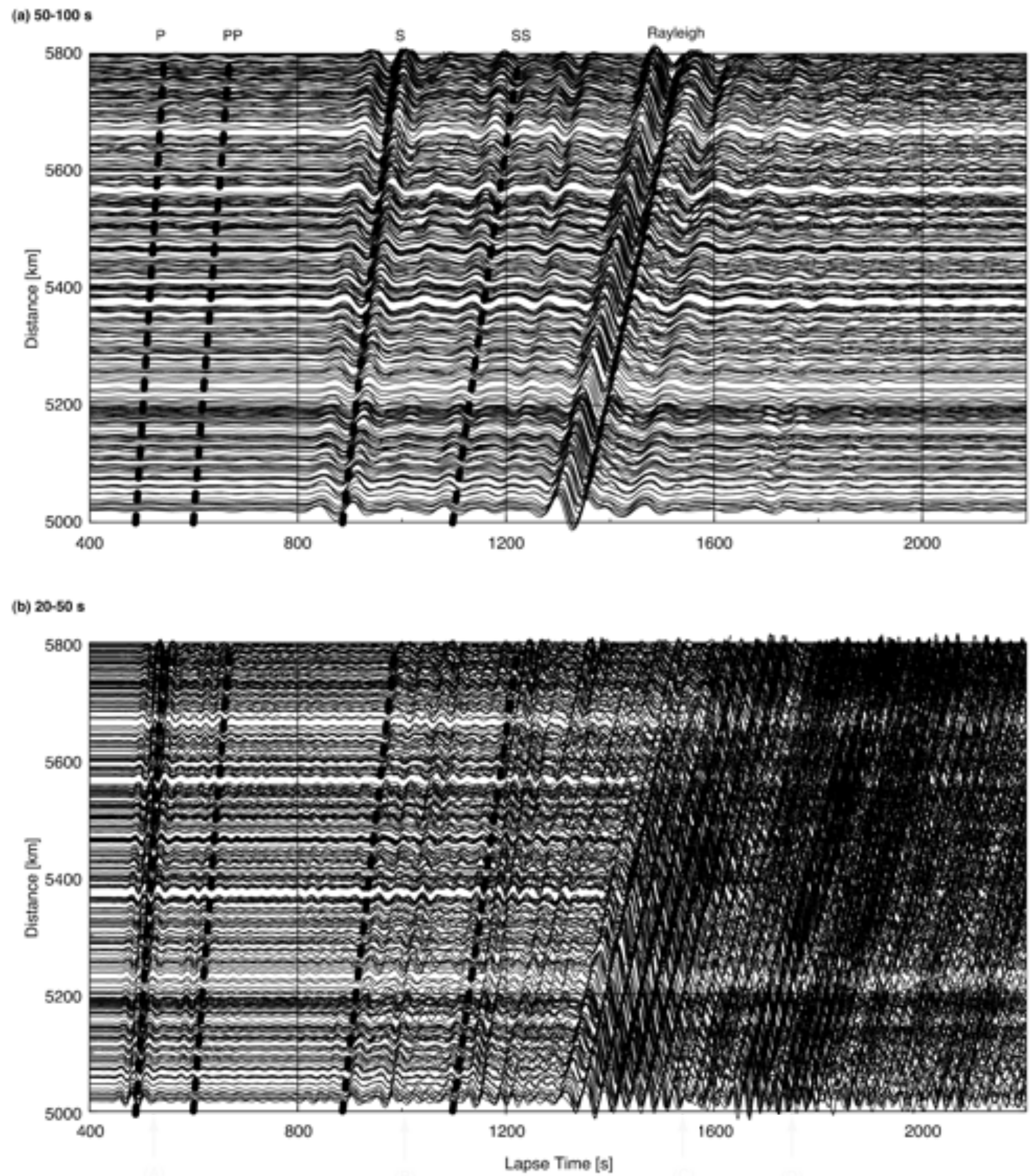


- Only relative tsunami height can be measured by pressure gauges
- Co-seismic seafloor deformation beneath stations results fictitious offset on tsunami
- Recent updates of data assimilation technique succeeded in separating between coseismic seafloor deformation and true tsunami height

Dense seismic observation

- Station separation ~ 20 km
- Targeting long-period band:
  - Wavelength ~ 100 km @ 25 s
- We can treat the traces as a **continuous wavefield**

- **Observation is only on the ground surface:** still difficult to assimilate to numerical models
- Data-driven approach: obtain more information from wave-field modeling



(Maeda et al., 2011, JGR)

Seismic gradiometry

- Taylor series expansion of seismic wavefield

$$\underbrace{u^{\mathrm{obs}}(x_S, y_S, t)}_{\text{station}} \simeq \underbrace{u(x_G, y_G, t)}_{\text{grid point}} + \left.\frac{\partial u}{\partial x}\right|_{x_G} (x_S - x_G) + \left.\frac{\partial u}{\partial y}\right|_{y_G} (y_S - y_G)$$

- Estimation of **wave at grid point** and **spatial gradients** by the least square

$$\begin{pmatrix} u_1^{\mathrm{obs}}(x_{S1}, y_{S1}) \\ u_2^{\mathrm{obs}}(x_{S2}, y_{S2}) \\ \vdots \\ u_N^{\mathrm{obs}}(x_{SN}, y_{SN}) \end{pmatrix} = \begin{pmatrix} 1 & x_{S1} - x_G & y_{S1} - y_G \\ 1 & x_{S2} - x_G & y_{S2} - y_G \\ & \vdots & \\ 1 & x_{SN} - x_G & y_{SN} - y_G \end{pmatrix} \begin{pmatrix} u(x_G, y_G) \\ \partial_x u(x_G, y_G) \\ \partial_y u(x_G, y_G) \end{pmatrix} \longrightarrow \mathbf{u}^{\mathrm{obs}} = \mathbf{G}\mathbf{m}$$

(Spudich, 1995 JGR; Liang and Langston, 2009 JGR)

- Inverse problem at each grid, however it only depends on station layout
  - Pre-computation of the kernel save the computational cost

Wavefield characterization

- **Divergence & rotation vector** with free surface B. C.
  - Convert derivative wrt depth to that wrt horizontal directions by B.C.

$$\mathrm{div}\,(\mathbf{u}) = \frac{2\mu}{\lambda + 2\mu}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) \simeq \frac{2}{3}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) \quad \mathrm{rot}\,(\mathbf{u}) = \left(2\frac{\partial u_z}{\partial y}, -2\frac{\partial u_z}{\partial x}, \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)$$

(Shapiro et al. , 2000, BSSA)

- **Slowness estimation**
  - observation = (amplitude term) x (propagation term)

$$u\,(x,t) = G\,(x)\,f\,[t - p\,(x - x_0)]$$

$$\frac{\partial u}{\partial x} = \frac{(\partial G/\partial x)}{G(x)} u(x,t) + \left[-p + \frac{\partial p}{\partial x}(x - x_0)\right]\frac{\partial u}{\partial t}$$

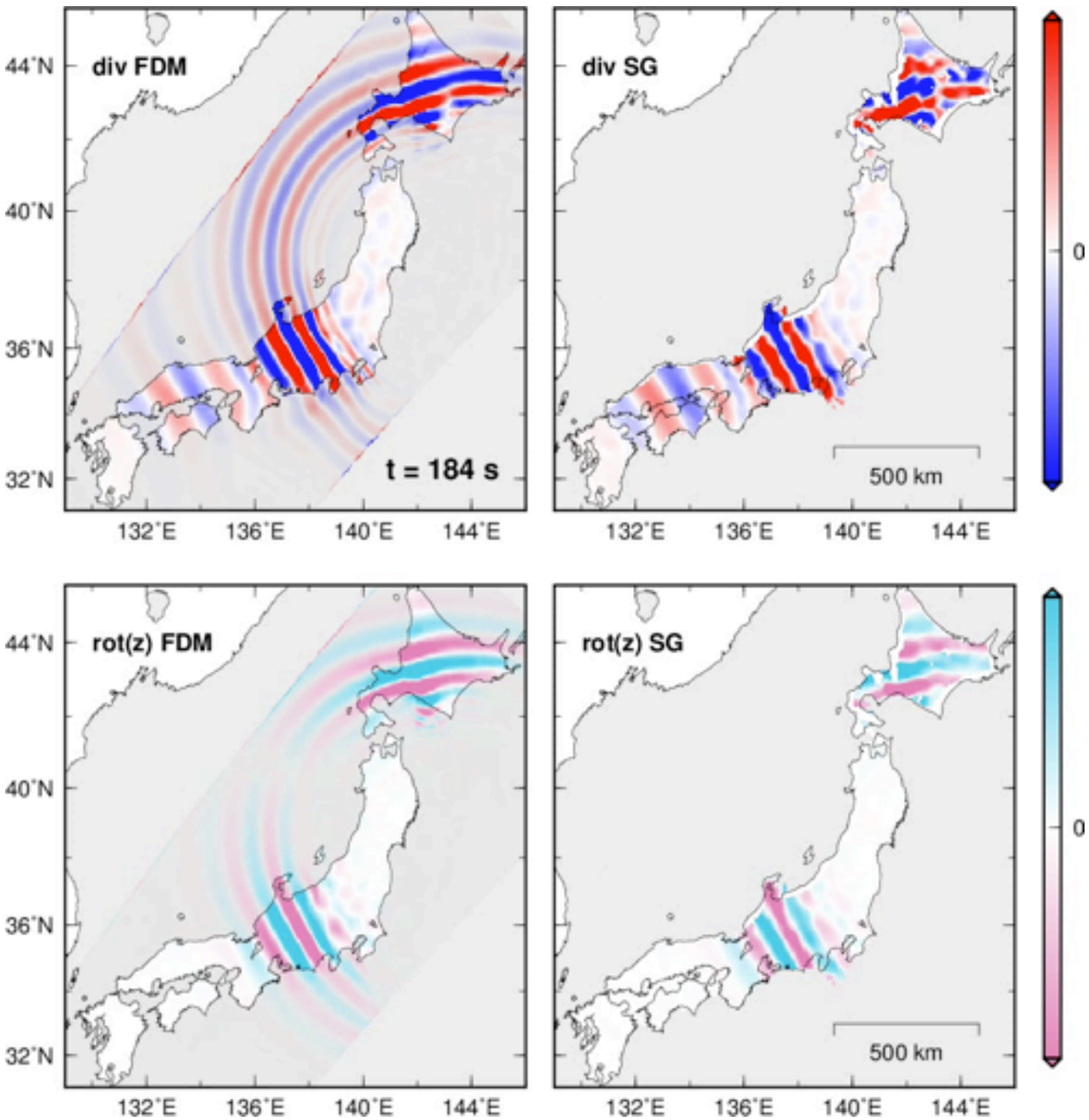
$$= A(x)u(x,t) + B(x)v(x,t)$$

  - ***A*(*x*)**: Term related to geometrical spreading and/or radiation pattern
  - ***B*(*x*)**: Slowness (arrival direction & phase speed)

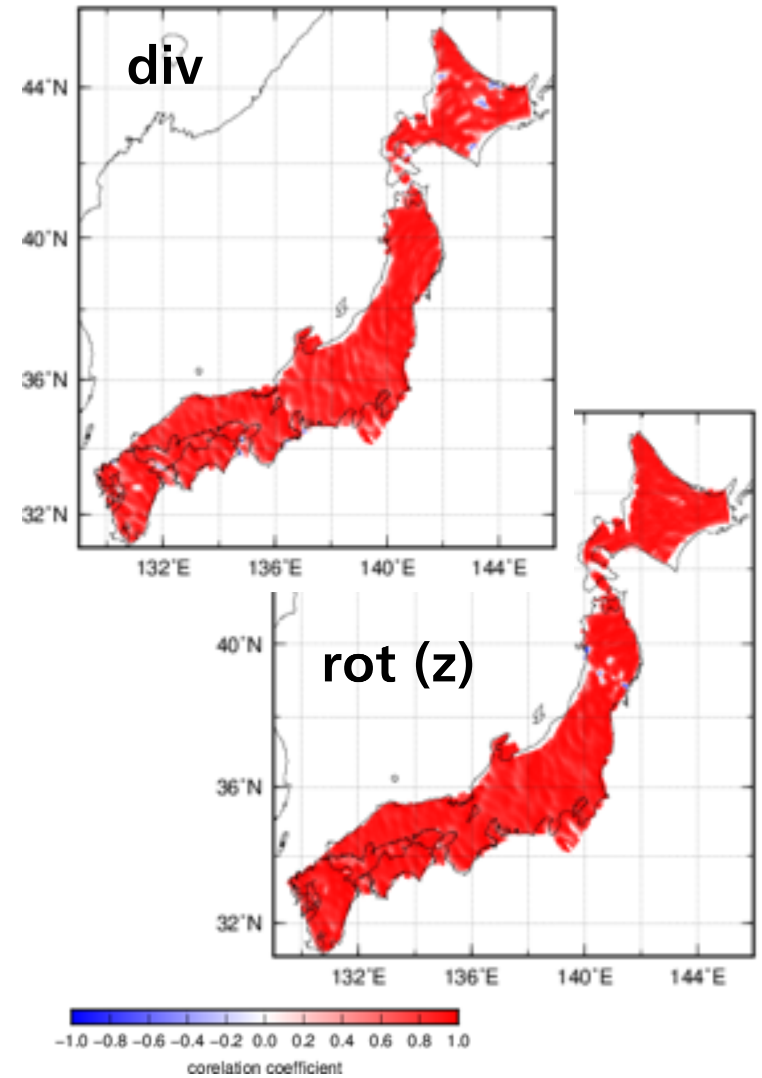
(Langston, 2007, BSSA)

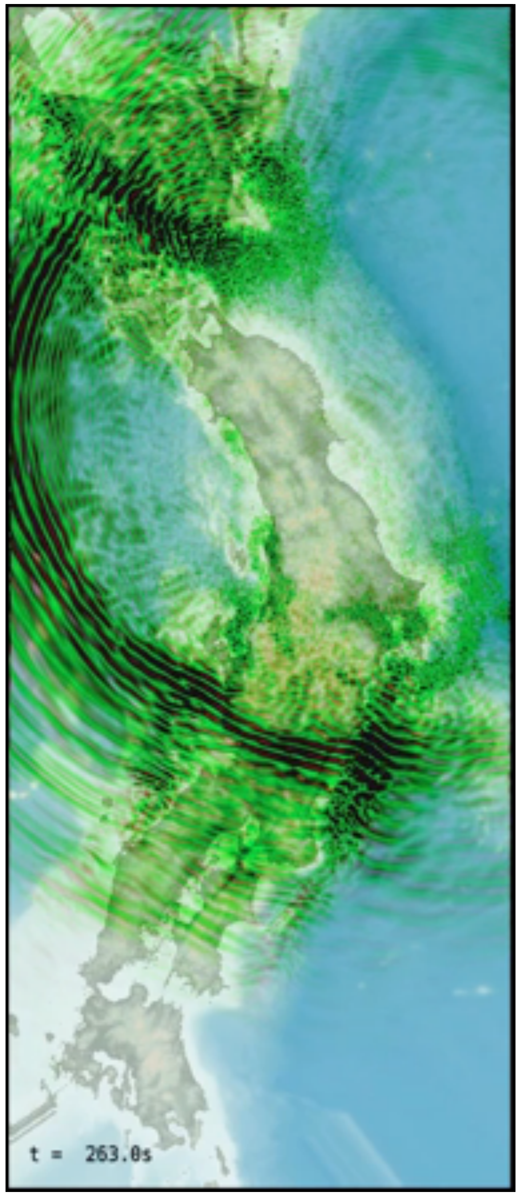
Synthetic test

div/rot decomposition@25-50 s



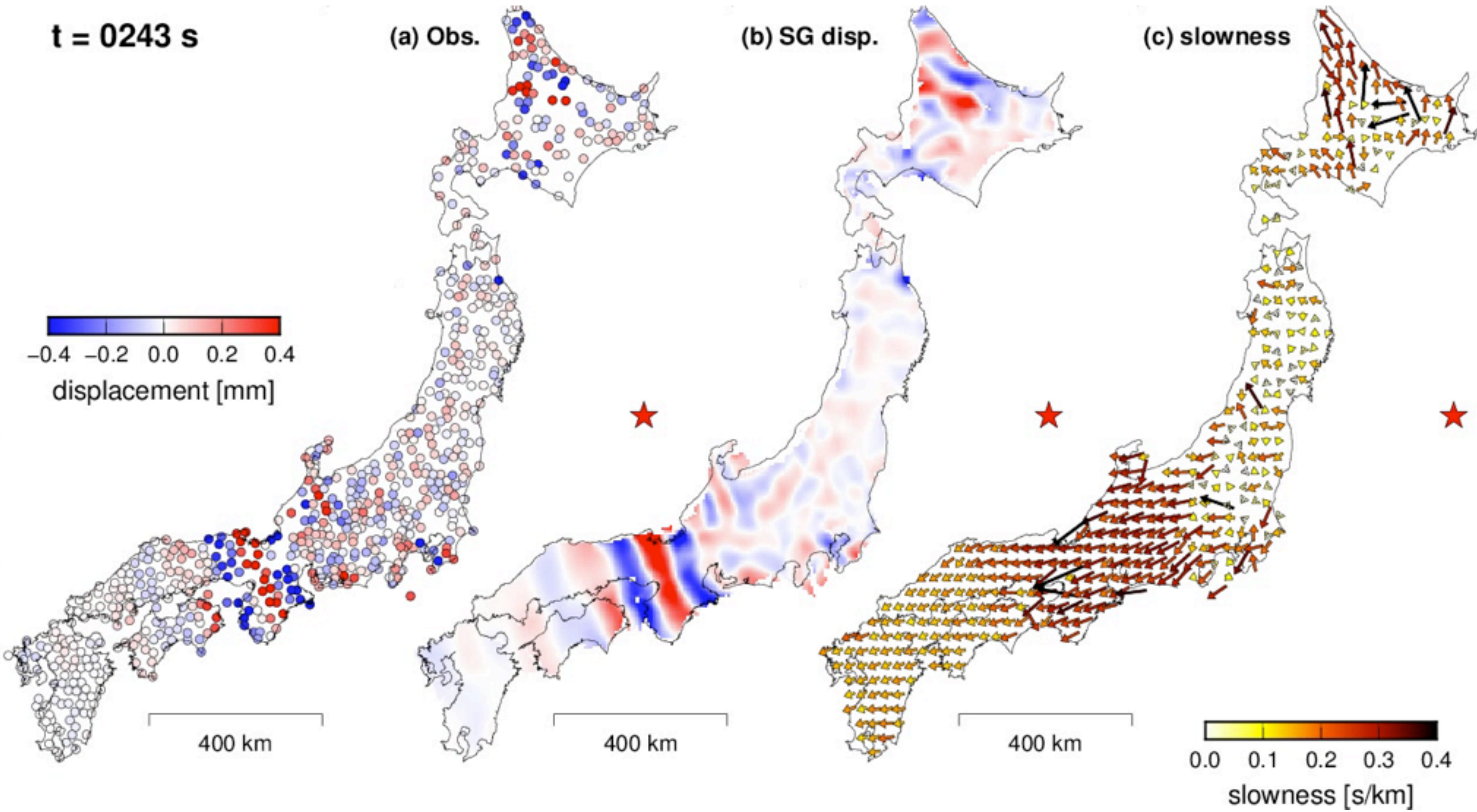
Goodness-of-Fit





- Hi-net with SG act as div&rot-seismometers
- Love & Rayleigh decomposition

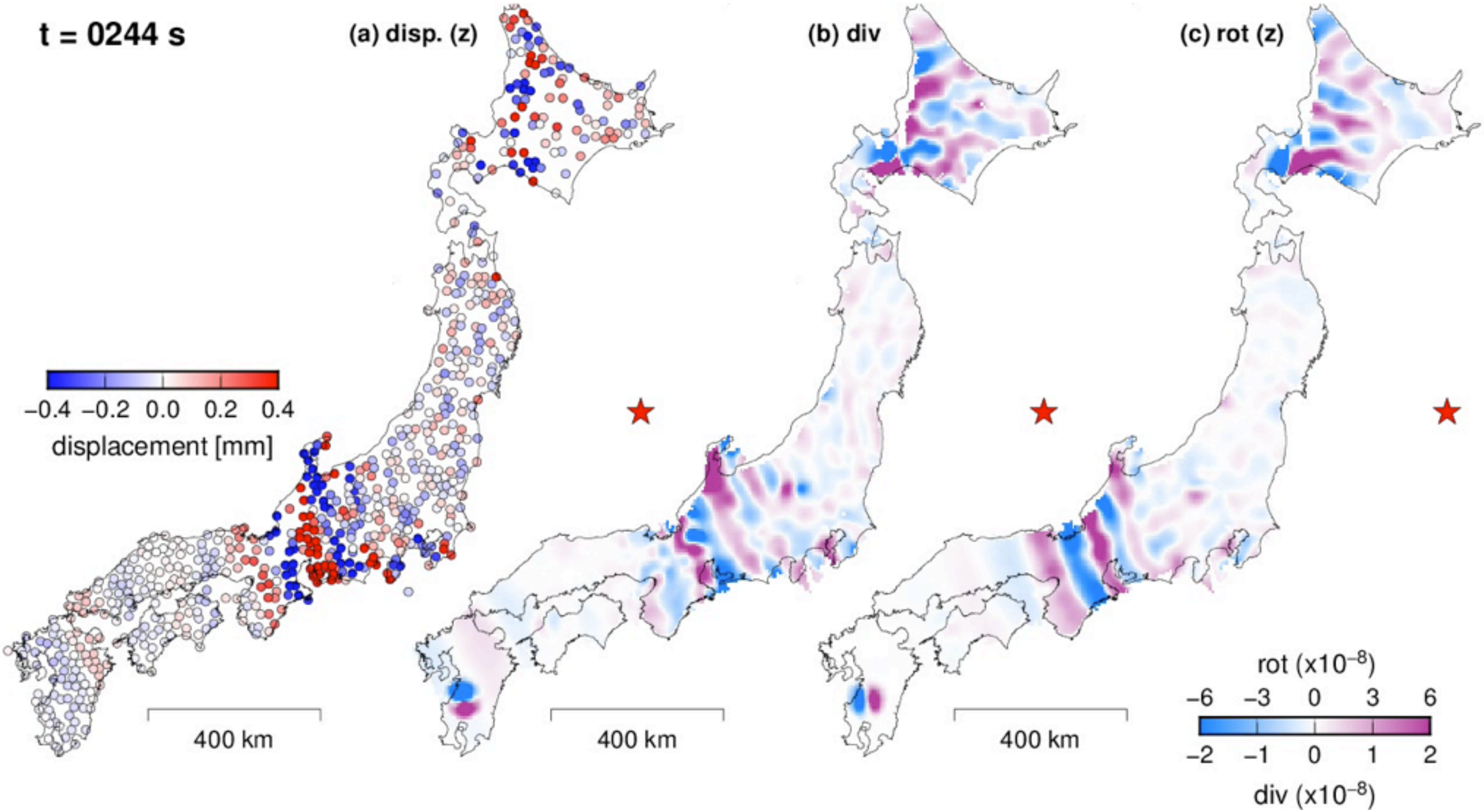
Example: 2005 Off-Tohoku outer-rise eq.



- In-situ estimation of slowness vector (speed & direction)

(Maeda et al., submitted)

Divergence & rotation decomposition



- Decompose the vector seismic wavefield into divergence (P&Rayleigh) and rotation (z) (SH&Love)

(Maeda et al., submitted)

Concluding remarks

- The full utilization of recent dense arrays enables us to track seismic/tsunami waves as spatially continuous wavefield
- Space-time visualization helps deep understandings of complicated wave phenomena
- Spatial wavefield is not only the simple visualization but is a target of data analysis:
  - Seismic gradiometry
  - Data assimilation
- Potentially useful for next-generation EEW?
- Next challenge: Assimilation of seismic waves ?

Acknowledgement: We used Hi-net records provided by National Research Institute for Earth Science and Disaster Resilience.